International Journal of Interactive Multimedia and Artificial Intelligence, Vol. 8, Nº6

The Game Theory in Quantum Computers: A Review

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Received 6 July 2021 | Accepted 26 July 2023 | Early Access 19 September 2023

Abstract

Game theory has been studied extensively in recent centuries as a set of formal mathematical strategies for optimal decision making. This discipline improved its efficiency with the arrival, in the 20th century, of digital computer science. However, the computational limitations related to exponential time type problems in digital processors, triggered the search for more efficient alternatives. One of these choices is quantum computing. Certainly, quantum processors seem to be able to solve some of these complex problems, at least in theory. For this reason, in recent times, many research works have emerged related to the field of quantum game theory. In this paper we review the main studies about the subject, including operational requirements and implementation details. In addition, we describe various quantum games, their design strategy, and the used supporting tools. We also present the still open debate linked to the interpretation of the transformations of classical algorithms in fundamental game theory to their quantum version, with special attention to the Nash equilibrium.

Keywords

Nash Equilibrium, Polynomial Time Quantum Problems, Quantum Computing, Quantum Game Strategies, Quantum Game Theory.

DOI: 10.9781/ijimai.2023.09.001

I. Introduction

The application areas of quantum computing cover disciplines such as chemistry, physics, artificial intelligence and data mining, among others. Currently, the most relevant studies are related to the field of cybersecurity, with the aim of verifying if classic cryptographic systems are robust enough to face quantum computer attacks. However, other applications in economics, finance, geopolitics, psychology, and even human behaviour, are drawing the attention of the research community. In these disciplines, it is common to use classic algorithms for decision-making and payment strategies, which are intrinsically associated with game theory.

Game theory, or *interactive decision theory*, is considered the formal technique for decision making. Although previously, some authors began their research around its formal outline, it was not until 1944 when Von Neumann and Morgenstern [1] proposed a mathematical structure based on set theory, prepositional logic, matrix algebra, linear geometry and group theory. The incipient study on the transformation of classical to quantum algorithms, and their potential entanglement in multiple strategies, has also been involved in the advancement of quantum game theory. However, the difficulty of these transformations sometimes resides in the design restrictions of quantum circuits, defined by DiVincenzo in his article [2]. These restrictions not only force the initialization of the game scenarios in a different way compared to their classic version, but they also affect the way the game is played as time evolves. During the game, there are player-related movements that are difficult to reproduce. In the real world, any given environment or *scenario* is continuously changing as the pursued strategies evolve.

In this work, we review some of the main studies related to quantum game theory algorithms and the different interpretations in its transformation from its classical orchestration. In addition, we address the main techniques and procedural background used in this area of knowledge. In this context, the present work complements other review efforts on the subject, such as that carried out by Guo *et al.* [3].

A. The Game Theory and Computer Science

Game theory, or the discipline associated to the *search for optimal decision strategies to maximize profits*, has shaped other areas of knowledge, mainly Mathematics and military strategy. Originally, games were adapted to the latter, since choosing an appropriate strategy provides an advantage over other opponents. The origins of games such as chess in the 18th century in Prussia, served as a means of teaching future army officers concepts assimilable to military tactics [4]. Throughout the centuries, we find examples of these tools such as the Scytale used during the war between Athens and Sparta 431 BC, the Caesar cipher used 100 BC, or Enigma used in WWII by the German army. However, the best example of a military communication instrument is, perhaps, Arpanet, created by the United States Department of Defense in the middle of the Cold War with the Soviet Union, which forged the foundations of the current Internet. That is why, through the centuries, we observe that the application of game theory has been supported with the cutting-edge technological tools and knowhow of each era.

Please cite this article as:
R. Pérez-Antón, J. I. López Sánchez, A. Corbi Bellot, "The Game Theory in Quantum Computers: A Review", International Journal of Interactive Multimedia and Artificial Intelligence, vol. 8, no. 6, pp. 6-14, 2024, http://dx.doi.org/10.9781/ijimai.2023.09.001

In the 20th century, the first programmable processors were also applied to game theory given the increase in calculation speed. For instance, C. Shannon proposed chess as a testing ground for the development of artificial intelligence in 1950 [5]. In 1996, G. Kasparov, the world's best chess player at that time, was defeated by the Deep Blue supercomputer. This piece of technology, created by IBM, was capable of forecasting 100 million plays per second.

Nevertheless, it is widely known that classical Turing Machines (TM) suffer from computational limitations [6],[7]. Currently, TM is accepted as the correct formalization of the algorithm concept. An algorithm is a sequence of executable logical steps that allow solving a problem. It must also meet two properties: have a finite description and be made up of discrete operations that can be mechanically executed [8]. Briefly, we will define TM as a theoretical concept with a series of deterministic mechanical execution steps on an infinite tape on both sides with read/write head. These execution steps are defined and collect all the computational finite automaton processing of the input string. The TM continues to perform execution steps until it reads a symbol for which no action is defined. Whenever a TM accepts an input string the machine stops. However, a string is not accepted if it is stopped in a non-accepting state or by infinite looping (it never stops). In the latter case we cannot know if TM rejects the chain. A TM computes *f*, where *f* is a decision problem (not a function problem), in time $T(n)$, where $n \geq 0$ finite if its computation in each input string x needs at most $T(|x|)$ steps.

That is why this hypothetical machine serves as a measurement tool to determine the limitations and complexities that can be addressed by classical computers. There are variations to the original TM for solving problems such as the multitrack variation, with bounded memory register, multi-tape, non-deterministic, *or the quantum TM.* All of them measure the computational limitation of an algorithm, in time and resources.

B. Decision Problems in Classical Vs. Quantum Computers

Decision problems belong to the formal mathematical realm of game theory. That is why any algorithm with the aim of solving this type of problem can be said to belong to game theory or interactive decision theory. As stated above, the execution of an algorithm has computational limitations, therefore, the algorithms for solving games or making decisions will also have them.

The theory of complexities or computational limitations were introduced in several articles by Hartmains and Stearns in 1965 [9]. Computational complexity is responsible for analysing the resources, time, and memory to solve a problem. The main objective of the computational complexity theory is to identify the processing limits. That is why the analytical comparison between limitations of classical and quantum computers is essential. In a classical computer, both low-level circuits (hardware) and high-level programs (software) act under a structure based on algorithms that solve problems iteratively (step by step). Finding the algorithm that efficiently solves a problem is synonymous with finding the minimum consumption of time and resources. That is why the scientific community has determined that algorithms that are solved in polynomial time are efficient and in exponential time are intractable. Although some intractable decision problems, it seems that they are possible to be solved by quantum processors. The mathematical notation for representing spatial and temporal complexity when $n \to \infty$ in the worst case is defined as O (big O-notation). The scale of complexities is defined as (1):

$$O(1) \subset O(lg\, lg\, n) \subset O(ln(n) \subset O\,(lg^{a>1} n) \subset O(\sqrt{n}) \subset O(n) \subset O\,(n\, lg\, n) \subset O(n^2) \subset \cdots \subset O(n^{a>2}) \subset O(2^n) \subset O(n!) \subset O\,(n^n) \quad (1)$$

TABLE I. Response Time for Two Values of the Size and Complexities n^3 and 2^n for A Step Value = 0.1 Millisecond

Complexity	$n = 32$	$n = 64$
n^3	3 secs	26 secs
2^n	5 days	$25 * 10^6$ years

If $O(g)$ is the asymptotic upper bound of the complexity of any algorithm and c is a positive constant of factors external to the algorithm, such as the machine to be executed. We have, as it is shown in Fig. 1, the polynomial time is the temporal complexity function $f(x)$, where x is the number of algorithm step value instructions. Therefore, the function f belongs to the complexity class of g ($f \in O(g)$ if there exists a c and an x_0 such that for all $x \geq x_0$ we have $|f(x) \leq c\,|\,g(x)|$ [10].

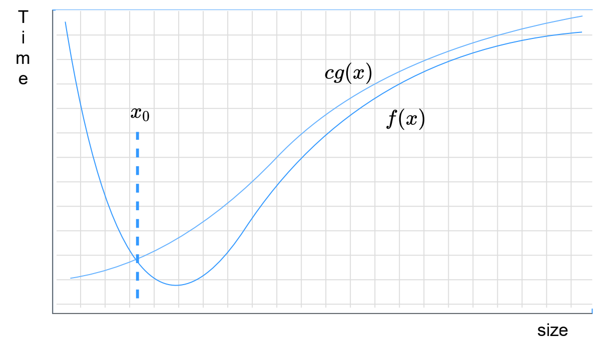

Fig. 1. Generic graphical representation of the comparison of functions in the calculation of complexities of any algorithm [11], including the problems generated by game theory.

Therefore, thanks to set theory, the characteristics common to all those decision problems with resolution in polynomial time and common characteristics can be grouped. The grouping of this type of mathematical problems according to their computational complexity or computational limitational are called *classes*, and their interrelation can be clearly seen in Fig. 2. In this way, we define some of the classes of fundamental decision-type problems based on their level of computational complexity:

P problems can be solved in polynomial time (linear, quadratic, cubic, etc.) in a deterministic MT. That is, the total time required by a processor to solve a problem that is bounded by a polynomial as a function of the size of the input and the number of configurations of its output.

NP problems are defined in a polynomial time. NP problems can be found in graph theory such as isomorphism or Hamiltonian paths.

PP The problems are solved in probabilistic polynomial time measured in a probabilistic MT. That is, the result obtained has an error with a probability of less than ½ for all cases. An example of an algorithm solved under probabilistic polynomial time is the Solovay-Strassen test [12].

BQP are problems that can be solved in polynomial time in a *quantum* TM. That is, the total time required by a quantum processor to solve a problem based on the size of the input (number of qubits), the number of configurations of its output and a maximum 1/3 probability of error for all instances and, therefore a success of 2/3. For example, we can find Shor's integer factorization algorithm in this class of problems [13].

BPP problems can be solved in probabilistic polynomial time measured in a probabilistic TM. That is, the result obtained has an error with a probability 1/3 and a success of 2/3. This type of problem is opposed to the Knapsack problem (KP) of combinatorial organization [14] solved by BQP, and all its elements must be included in the proposal for its resolution.

There are more kinds of decision problems such as NP-Complete, EXPTIME, L or NL, although we will not address them in this work. The class PH was determined by Larry Stockmeyer [15], and it unifies all the classes of hierarchical polynomial complexity. The potential of quantum computing is that, according to Aaronson [16], there is some evidence that BQP is not contained in PH. This entails that we could be approaching the resolution of exponential problems, intractable until now by classical computing.

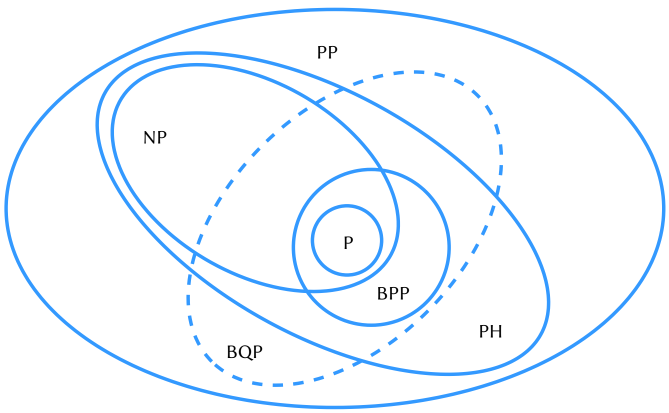

Fig. 2. Diagram of relationships between the different classes of complexities [17]. The BQP class can solve some problems in polynomial time that are not contained in PH.

C. Decision Strategies in Game Theory

Game theory has its own defined and structured decision problems according to its peculiarities. The types of games, their decision strategies, the format of the scenario and the number of participants are some of the characteristics of classical modelling. These strategy models correspond to a specific mathematical problem included in polynomial or exponential classes. Therefore, the correct initial definition of the classical strategy will be transformed into a class of computational complexity or limitation defined by both the quantum and the classical realms. Some of these strategies and payoff models are essential for understanding the transformation from classical to quantum algorithms.

Cooperatives and Non-Cooperatives strategies are characterized by the realization of alliances between the players with the objective of intensifying the maximum common benefit. These types of strategies can be found applied in common population resources such as a recycling plant, desalination plants, fire brigades, etc. Where the contribution of each player amplifies the benefit obtained by all participants. These cooperative strategies can also be used in areas such as politics, geopolitics, economics, armed conflicts, national and international markets. On the contrary, non-cooperative strategies are defined as those used by each player with the objective of satisfying individual benefit.

Sum 0 is closely linked to the interdependence between payments. Taking as the absolute factor payment to be distributed among all the players, a 0-sum game is understood to be one that the benefit of one player affects the losses of all the others. In other words, what a player has won necessarily comes from what another player or players have lost. This concept is very widespread in the financial world since the pie to be shared is finite and whenever an investor in the stock market gains the profit is associated with the loss of another individual. The opposite concept is *non-zero-sum games*. They are defined as those in which the cooperation between the participants of the game generates an equal and common benefit or loss.

Nash equilibrium was theorized by John. F Nash, who was later awarded the Nobel Prize in Economics in 1994 for his equilibrium analysis in non-cooperative game theory developed in 1950 [18]. The Nash equilibrium in game theory is becoming the most prominent unifying theory in the social sciences as indicated in his article [19]. Nash introduced the concept based on the relationship between the strategic equilibrium of the players and their maximum profit [20], which reads: "any n-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the $n-1$ strategies of the other players in the countered n-tuple". A self-countering n-tuple is called an *equilibrium point.* In this strategy, it is assumed that all players know each other's strategies and do not cooperate with each other. In addition, the best strategy of a player is not synonymous with maximum payout but with less loss or 0 losses. Other essential concepts for understanding the Nash equilibrium are pure or mixed strategies. The pure strategies are those that each player chooses with probability 1, as an example we have the game of *rock, paper, and scissors.* In this game, each player selects his strategy based on a single payment. However, if we assign to each pure strategy a probability on the payout, we will be defining the mixed strategies. Mixed strategies are a generalization of pure strategies, therefore, in each one we can find a pure one. Nash showed that any finite rectangular game has at least one Nash equilibrium in mixed strategies [20].

Pareto optimal is one of the fundamental theories of welfare economics and was introduced by Vilfredo Pareto in 1896 [21]. And it is currently applied in different areas such as operations research, decision making, optimization with multiple objectives or cost-benefit analysis. It consists in that, given an initial allocation of earnings among a set of players, a change towards a new allocation that at least improves the situation of one individual without making the situation of the others worse is called *improvement.* An allowance is defined as *Pareto optimal* when no further improvements can be achieved. Therefore, it is no longer possible to benefit more individuals in a system without harming others. The *Pareto frontier* is identified with the function $f(x)$, where when expanding its domain, the gain of an individual is a consequence of the decrease of another participant. We formally define the concept as: let P be a multi-objective optimization problem, then a solution P_i is the Pareto optimal when there is no other solution P_j such that it improves on one objective without worsening at least one of the others.

Stochastics games were originally devised by Shapley [22]. They consist of achieving different states of the game system in time. That is, over time the choice of strategies of the players are conditioned to the current state or set of variables. Fundamental examples of these games are dice-based. They could be treated as games where the chance or different variables change the players' choice of strategies and payout over time. This game model has its application in market economies, such as the stock market. Furthermore, stochastic games can be approached from different perspectives, such as finite or unlimited in time, with partial information to the players, non-cooperative or 0-sum, among others.

D. Characteristics of Quantum Computers

Quantum computing is a different computing paradigm from digital computing. The internal logic architecture of a classical computer works with electrical pulses that are translated into high voltage 1 and low voltage 0, with deterministic input and output. However, in a quantum computer the conceptual change of its internal structure is based on quantum mechanics, not deterministic. Therefore, it is essential to review some of these concepts to get closer to this new computational model.

The main characteristic of quantum processors lies in the ability to manipulate quantum bits, known as *qubits*. Qubits can be represented by subatomic particles like electrons or photons with the intrinsic characteristics of quantum mechanics. That is why quantum computers use the properties of entanglement, superposition, and parallelism to optimize computational processing. The concept of qubit is not associated with a specific physical system, and they are described as a unit module vector in a complex two-dimensional vector space. The two basic states are are $|0\rangle$ and $|1\rangle$, but qubits can also be found in a state of superposition [23].

The superposition is associated with each physical system, where there is a Hilbert space (H) known as the state space of the system. The system is completely described by its state vector (represented in (2)), which is a unit vector in that state space. The states of the qubit represent a vector of states in a vector of states in the Hilbert 2-D space (H^2) with an orthonormal base.

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \tag{2}$$

Therefore, a qubit represents the conjugated states with the complex numbers α and β, defined as (3):

$$|\psi\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha|0\rangle + \beta|1\rangle \tag{3}$$

If α and β are not null, we could describe these factors as the 0 or 1 probability of the state representing a superposition as (4):

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\frac{|+\rangle + |-\rangle}{\sqrt{2}} + \beta\frac{|+\rangle - |-\rangle}{\sqrt{2}} \tag{4}$$

This means that unlike the classic bits, qubits can have both states at the same time (0 and 1). On the other hand, quantum entanglement only occurs between two or more qubits generating a unique state of the system. This intrinsic characteristic of the particles, without similarities in classical theories, is known as the ERP paradox due to its prediction in 1935 by Einstein, Podolsky and Rosen [24]. For our interest in this work, we will only focus on the characteristic of the combination of the quantum states of one or more qubits. The maximum entanglement between two qubits is called the Bell state [25] and its mathematical notation is (5):

$$|\psi\rangle = \frac{|01\rangle - \langle 10|}{\sqrt{2}} \tag{5}$$

where, the possible states of $|\psi\rangle$ are $\{\alpha\,|\,00\rangle, \beta\,|\,01\rangle, \gamma\,|\,10\rangle, \delta\,|\,11\rangle\}$, and α, β, γ, δ are the probabilities each state. This quantum capacity probably increases to infinity depending on the number of entangled qubits. One of the singularities of this formulation is the capacity for the continuity of entanglement of the particles even when they are separated by millions of kilometres. Furthermore, with the feature of parallelism there is the possibility of simultaneously representing the values 0 and 1. Quantum algorithms that operate on superposition states, simultaneously perform operations on all combinations of the inputs. This is where the potential of quantum computers resides.

Another fundamental question in the characteristics of quantum computers is the transformation of the states of the qubits. Currently the technologies used for handling qubits are based on superconducting circuits, ion traps or photonic circuits. These complex physical structures have as their fundamental objective to deliberately modify the states of the qubits. These changes generate the algorithms programmed to yield the desired results of the quantum processors.

The evolution of a closed quantum system is described by a unit transformation [26]. This is that state $|\psi\rangle$ from the system to time t_1 is linked to the state $|\psi'\rangle$ at the time t_1 by a unitary operator U that depends only on t_1 and t_2 such that $|\psi'\rangle = U|\psi\rangle$. A unitary operator U is neither more nor less than a matrix, therefore, applying U on a state is to operate the system by the matrix U. It follows that the state $|\psi'\rangle$ will be determined by the application of a unitary operator (a matrix). As can be seen in Fig. 3, U_f is the unitary operator applied to the state $|\psi\rangle$ and throws us as a result $|\psi'\rangle$:

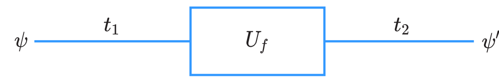

Fig. 3. Unitary operator. Graphical representation of the transformation of states $|\psi\rangle$ on timeline t, when applying a unitary operator U_f.

Therefore, like classical logic gates, state operators modify the states of the qubits, although with some differentiating characteristics over the classical ones. Basically, a quantum gate is a unitary matrix, which, when applied to the qubits, performs a state transformation. The combination of the quantum gates together with the control artifacts generates the unit operators that make up the quantum circuits. Next, we detail the generator of entanglements.

The Hadamard gate can only be applied to a qubit, and its main function on the application of a qubit is for state 0, $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and for state 1, $H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. It is also defined in matrix form as (6):

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} \tag{6}$$

We can observe in (8) the internal states of a qubit by applying the Hadamard gate to a state $|0\rangle$ and $|1\rangle$. These states (7) are not maintained in the measurement of the qubit, but collapse to 0 or 1.

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}; \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} \tag{7}$$

This represents the quantum Fourier transformation [27] rotating π about the z axis followed by a rotation of $\frac{\pi}{2}$ about the y axis. In addition to having the characteristic of generating a Bell state, that is, interlacing and deinterlacing qubits in the way described in (8).

$$\begin{matrix}|0\rangle \;—\boxed{H}—\; \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\\ |0\rangle \;—\boxed{H}—\; \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\end{matrix} = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) \tag{8}$$

where gate H_4 becomes 4×4 matrix represented in (9):

$$\mathrm{H} = \begin{pmatrix}1 & 1 & 1 & 1\\1 & -1 & 1 & -1\\1 & 1 & -1 & -1\\1 & -1 & -1 & 1\end{pmatrix} \tag{9}$$

There are different quantum gates with different objectives or functionalities. For example, the Pauli gate X will exchange the states of a qubit, like the binary NOT gate, that is, if initially the state is $|0\rangle$ it will transform it to a state $|1\rangle$. On the contrary, the Pauli gate Z involves the state $|1\rangle$ exchanging it for -1, leaving the amplitude (probability) of the state $|0\rangle$ untransformed (Fig. 4).

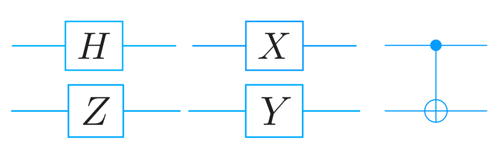

Fig. 4. Graphical representation of quantum gates. H - Hadamard, X - corresponds to a 90° rotation on the x axis, Z corresponds to a 90° rotation on the z axis, Y - corresponds to a 90° rotation on the y axis. In addition to the representation of CNOT.

However, one of the negative characteristics of quantum processors is their fragile stability in state conservation. This conservation of states in a processor is known as quantum coherence, and it can be defined as the conservation of the state of a system in superposition with time. This coherence is physically sensitive to interference from the environment, and can be destroyed by vibrations, electromagnetic disturbances, and other circumstantial disturbances such as sounds, earth vibrations or adverse weather effects. That is, the particle collapses in a state as if it were being measured, losing the multistate characteristic. The destruction of quantum coherence is what is known as *decoherence*.

The coherence time in a quantum computer is essential for the correct implementation and obtaining the expected results. That is why scientific research in the field of quantum computer implementation is vital in advancing in this area. Hadamard gates enhance the intrinsic characteristics of qubits entanglement, therefore, not using them makes a quantum computer become a reversible classical processor.

II. Design of Quantum Games

The conceptual paradigms of quantum computing have been formalized and the parameters established in classical game theory. In addition to demonstrating the potentiality in quantum computational efficiency. Therefore, we are now able to unify both concepts to know the theory of quantum games.

The decision-making strategy is related to the environment and the actors involved. If, on the one hand, on the environment model we can differentiate symmetric and asymmetric games, zero sum, with or without Nash equilibrium and Pareto optimal, combinatorial, perfect information, stochastic or differential. On the other hand, according to its actors, we find cooperative, non-cooperative or multiplayer games, among others. However, not all these classic game theory models can be applicable to the design of circuits and quantum algorithms, given their complexity and lack of research. Consequently, a little explored field of research opens here.

A. Requirements in the Design of Quantum Games

According to DiVincenzo, quantum circuits must have the following requirements for their construction:

- a scalable physical system with well characterized qubits,
- the ability to initialize the state of qubits to 0,
- long relevant decohere times,
- a universal set of reversible quantum gates, and
- a specific measurement capability of the qubit.

We will not discuss requirements 3 and 5, since we consider theoretically that the circuit is stable and without decoherence, in addition to generating an appropriate final measurement. But we will add some specific characteristics of quantum circuits, essential for understanding their design:

- A quantum algorithm is iterative, it is not possible to develop loops.
- Quantum states cannot be cloned, there are no FANOUT circuits that can replicate qubits.
- Quantum circuits are inherently parallel, allowing a function $f(x)$ to be evaluated for multiple values of x simultaneously.

On the other hand, we will need a valid justification for the transformation from classical to quantum algorithms. Well, according to Wu et al. [28], it must be considered that a quantum computer without Hadamard gates is essentially a reversible classical computer and therefore, we cannot consider significant the circuits implemented without these unit matrices of game theory classics. Furthermore, if we add to these requirements and characteristics the three principles of quantum games that they develop [29], we have a technical challenge and an unknown computational complexity, since it is currently unknown how BQP compares with BPP.

Even so, there are laudable attempts to debate the classical models of game theory applied to quantum algorithms, such as the Nash equilibrium, that deserve all the attention given that they are generating the theoretical basis for the evolution of this area.

B. Quantum Nash Equilibrium

The historical example to define the Nash equilibrium in a classical system has been the Prisoner's Dilemma [30] where the desertion of each of the players implies the maximum individual benefit. This game also has the characteristic of establishing an example of pure strategies for each player, since chance does not intervene, and probability is not established. The concept of pure strategy was extensively studied by Antoine Cournot in his work on oligopolies [31] and we can consider it as a particular case of mixed strategies.

However, when implementing the Prisoner's Dilemma game in quantum format according to Van Enk Wu et al. [32], individualistic strategies are eliminated, transforming the game from non-cooperative to cooperative given their retrospective contemplation of both players of their strategies. That is, given the reversibility of the unit matrices, it would be possible to go back and optimize the strategy of both players to obtain the maximum payout. Furthermore, entanglement tends to be considered as a mediated communication [33] or a requirement function according to the theory of abstract economics [34], which does not correspond to the original classical game. The EWL quantification protocol [35] has so far been the most accepted for the quantum transformation of the Nash equilibrium. However, Van Enk Wu contradicts the idiosyncrasy of not preserving the non-cooperative game condition and therefore the elimination of the Nash equilibrium. There is an interesting discussion on this topic in [36].

C. Actors and Their Game Decision Strategies

In a non-cooperative game there are players, the setting and the rules of the game that comprise the strategies and the payouts. These components are defined in standard classical game theory [37] and quantum notation as a tuple $\langle N, \Omega, P \rangle$ where N is the number of players, Ω_j with $1 \leq j \leq N$ are the strategies of each player, and P_j the payoff function $P\colon \Omega_j \rightarrow R^N$ on each of the strategies. Therefore, the interaction between classical and quantum strategies is hypothetically possible.

This interaction can be seen with the payoff matrix in Table 2. Meyer demonstrated with the Penny-Flip game of two players with the zero-sum strategy, that a quantum strategy will always win over a classical one [38].

TABLE II. Matrix of Payments Between Quantum and Classic Strategies

Type of Actors	Classic	Quantum
Classic	(0,0)	(0,1)
Quantum	(1,0)	(1,1)

At first it might seem that a quantum strategy versus a classical one is always the winner, but Anand and Benjamin [39] demonstrated that a particular classical algorithm, such as the one proposed by Meyer, can beat a quantum one in the Penny-Flip game if it is generalized.

Despite everything argued so far, even in the predictive results of any classical game model, human behaviour breaks the mathematical formalism. Well, game theory assumes unrealistic levels of rationality of its players according to Chen and Hogg. If we translate this reality to the probabilistic results of quantum computers, then the initial payout matrix will change for each strategic decision of the players. The similarity in the decision making of a human player with the results of a quantum computer is demonstrated in their work. Where it seems to be verified that quantum entanglement is a rational human cooperative behaviour with the objective of obtaining the maximum benefit.

III. Quantum Games Implementation Techniques

The transformation from classical to quantum algorithms is a real effort given the requirements seen in Section 2. The applied implementation techniques stand out for their originality as the insertion of tertiary qubits – qutrits [40]. Although the debate centres on the insertion of Hadamard doors. The Hadamard unit matrices entangle the states of the qubits generating multiple overlays if they apply to more than one qubit. It is here where the power of quantum computers lies, and its parallelism seen in Section 3. Therefore, the techniques and results to design quantum games open the interpretive debate of the correct implementation.

Considering the limitations of computers and the types of classes of decision strategy problems seen in Section 1, we can affirm, looking at Fig. 2, that all decision strategy problems of type P can be solved by BPP which in turn are integrated into BQP. Therefore, in principle it seems that all decision strategies in game theory can be solved by a quantum computer. Consequently, there will be a computationally quantum resolution that satisfies the best game strategy for each participant or participants. However, the construction and design of these functions in a quantum computational architecture has yet to be solved for all decision strategy problems.

A summary of implemented quantum games can be found in [3] where the game and the quantum contributions that have been made are defined. In Table III, we expand some of these definitions of [3], including some of the technical characteristics used in their implementations. Some interesting techniques applied to other games are listed as well.

TABLE III. Structure and Implementation of Some Quantum Games

Game	Structure	Implementation
Prisoner's dilemma	Hadamard, without Hadamard, Multiplayer	Mathematical notation, Qiskit (IBM), Various unknown computers
Penny flip	Hadamard, without Hadamard	Mathematical notation, Various unknown computers
Five in a Row - Gomoku	Qutrits	Mathematical notation
Sudoku	Without Hadamard	Python
Poker TH	Hadamard, Multiplayer	Qiskit (IBM)
Bingo	Without Hadamard	Qiskit (IBM)
Monty Hall	Hadamard Qutrits	Mathematical notation
Battle of sexes	Hadamard	Mathematical notation
Rock–scissors–paper	Hadamard	Mathematical notation

Prisoner's Dilemma comprises different variants of the game. However, the most classic one is described by Albert W. Tucker who formalized the game on prison rewards [4]. It belongs to the group of zero-sum non-cooperative games, where the Nash equilibrium is determined according to the Pareto optimal strategy. The original game describes the situation of two participants where they ignore the decisions made by both. Two thieves (Alice and Bob) are caught by the police. Since the police do not have enough evidence to convict them, they propose a deal (summarized in Table IV):

- Alice and Bob confess to the crime -> Alice and Bob are sentenced to 6 years in prison.
- Alice or Bob confess the crime -> Whoever does not confess is sentenced to 10 years in prison and the one who has confessed to 1 year.
- Alice and Bob do not confess the crime -> Alice and Bob are sentenced to 1 year in prison.

TABLE IV. Matrix of Payments Classic Game Prisoner's Dilemma

	Confess	No Confess
Confess	(3, 3)	(-5, 5)
No Confess	(5, -5)	(-1, -1)

All the works on this game are implemented in mathematical notation, although it is worth highlighting the comparison made [30] on human strategies and quantum computers, or in [41] the duality map comparison. Others, such as [42] and [43], focus on classical transitions after analysing quantum decoherence and the null wave function. In addition, in the work [44], the multiplayer version is analysed. And in [44] the Hadamard entanglement gates are not used, although they are used in [45] [46] and [47] where the latter also analyses the applied unit matrices.

Penny flip is related to the flipping of a coin and obtaining heads or tails. However, the quantum strategy is added in player Q. It consists of player P placing a coin head up in an opaque box. After that, they will take turns (Q, then P, then Q) shaking the box or not. P wins when the coin is upside down when the box is opened [38]. This is a zero-sum strategy game for two that could be traditionally analysed using the following matrix reward (Table V).

TABLE V. Matrix of Payments Penny Flip

	NN	NF	FN	FF
N	-1	1	1	-1
F	1	-1	-1	1

The most outstanding works on its implementation and debate are [38] and [45]. However, it is worth highlighting the work [46] where an experiment is carried out intertwining four coins.

Five in a Row is original from Japan and known by different names in other countries. It consists of a 15×15 or 19×19 matrix where the players alternate in placing their chips on the squares. The winner is the player who manages to form a row, column, or diagonal with k of his chips, where k is the number of cells. The generalization or scalability of the game can be described as (m, n, k), where $m \times n$ will be the dimension of the matrix and k the number of continuous lines to get. In these typical games of the five in a row and Weiqi we can highlight the exotic implementation of [26] with qutrits [38].

Sudoku was popularized in Japan in 1986, although it is proven that the original creator was Leonhard Euler (1707-1783), by establishing the guidelines for the calculation of probabilities to represent a series of numbers without repeating incorporated in The Greco-Latin Square, Euler's Square or Orthogonal Latin Square. This game consists of a 9×9 matrix where the decimal numbers except 0 are placed in rows and columns. The challenge is not to repeat any decimal in the same row or column or 3×3 sub-matrix. The original matrix is initialized with some numbers that offer clues to start filling in the boxes. This is a single player game, and its initialization tracks must be at least 17 to have a single solution. This popular game is implemented in the high-level Python language without quantum entanglement.

Poker-TH is a multiplayer card game in addition to a mediator player. The mediator player exposes his cards, and the other players must decide the best combination between their cards and those of the mediator. The winning card combination is established from the beginning in a hierarchical range. However, the payoff matrix varies depending on the independent strategies. Each player in his turn bets on his combination, and the other players must accept the bet and continue the game or abandon and win nothing. Therefore, the payoff matrix varies according to the rational independent strategies. It is a game of sum greater than 0, however, in the individual strategies the Nash equilibrium is established, although not in the global game. This game has several variations and the most relevant are based on the number of cards to be dealt between the players and the limitations in the payout matrix. In [48], this game is implemented in IBM quantum computers, with the didactic goal of teaching quantum computing. In addition, it demonstrates decoherence and error mitigation techniques.

Bingo is a very popular game of chance. Players have numbered boxes, and the mediator randomly draws a number from the initially established range. The winning player must own all the numbers in his box from the previously drawn numbers. In this game, the combinatorics, and the number of boxes that each player has intervenes directly. The calculation through the hypergeometric distribution determines the probability (10) of the player to win:

$$P = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \tag{10}$$

where k is the number of the box, x the value of the variable or number of hits in each extraction, N the size of the sample and n the number of each extraction. Therefore, in a range of 90 numbers, the probability of winning on draw 65 is 0.45% for each box. This game is implemented without Hadamard gates of entanglement in the work of [49] on IBM quantum computers.

Monty Hall is taken from the 1975 US television contest *Let's Make a Deal* and has become a real mathematical problem of probability. The name was assigned referring to the presenter of this program and we can also find it as the Monty Hall paradox. The game consists of offering the player the choice of opening three doors. Only one of them hides the desired payment such as a car, a house, money, etc ... depending on the version of the game. Once the door has been chosen by the player, the moderator reduces the choice possibilities to 2 by eliminating a door and again offers the player a new choice. The debate on the game's payout probabilities began in 1990 through the journalistic columns written by Marilyn vos Savant [50], solving and demonstrating the theory that Steve Selvin introduced in 1975 [51]. The controversy lies in the change or not of probabilities. That is, at the beginning the payment of the game has a probability of 1/3 over the 2/3 of losing. However, by eliminating one of the gates we would understand that the probability is now ½ pay and ½ lose. Although if we consider the probabilities assigned at the beginning of the game, the best strategy is to select a new door since the probability of payment continues to be 2/3 (Fig. 5).

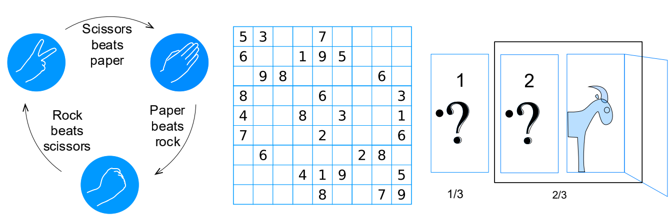

Fig. 5. From left to right: graphical representation of the games Rock-Scissors-Paper, Sudoku, and Monty Hall.

One of the proposals for the implementation of the game in quantum format by [52] is to take the beginning of the game as a three-dimensional Hilbert space where only the moderator reacts in a quantum way leaning on an interlaced Notepad or not, with the aim of saving the information. However, [53] they show that where both participants, contestant and moderator, have access to quantum strategies, the maximum entanglement of the initial states produces the same benefits as the classical game.

Battle of sexes is like the Prisoner's Dilemma, although in this case both participants have full information about payouts and strategies. It consists of selecting the best desired strategy and achieving the greatest benefit individually, therefore, it complies with Nash equilibrium. The initial approach consists of making leisure decisions for a couple (Alice, Bob) with the premise that they both want to be together currently. However, Alice prefers to go to the theatre and Bob wants to go to the movies. The payment matrix according to their preferences is shown in Table VI.

TABLE VI. Matrix of Payments Classic Game Battle of Sexes

		Bob	
		Theatre	**Movie**
Alice	**Theatre**	(2,1)	(0,0)
	Movie	(0,0)	(1,2)

In their article [54], Marinatto and Weber demonstrated that the use of entangled quantum strategies by both players does not improve the classic payoff matrix of the game and therefore generates the same resolution as the classic version of the game. However, a later article by Du *et al.*, [55] discusses [54] approach, proposing a different structure. Thus, Du implemented the game by applying mixed strategies for both players, where each player can freely choose his strategy. That is, they can apply entanglement or not, therefore, the transformation from classical to quantum game seems to demonstrate its efficiency.

Rock-Scissors-Paper was designed in China centuries ago. Today, it is an internationally famous game, and its rules are easy to learn. The game consists of two players simultaneously and in a single movement they determine their non-cooperative strategy. In this case, they determine their weapon which can be rock, paper or scissors. The payoff matrix is represented in Table VII.

TABLE VII. Matrix of Payments Classic Game Rock-Scissors-Paper

	Rock	**Paper**	**Scissors**
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(1, 1)

The original game ends in a single action and therefore a Pareto optimal strategy model results. Given that, if one player gets paid, the other gets nothing. However, there is the interpretation of the game with n repetitions. In this case, the game has only one Nash equilibrium and in each round the probability of payout becomes 1/3. Iqbal, in his article [56] studies this game in its repetition format, in an attempt to stabilize the evolutionary sequence (EES) on the Nash equilibrium by applying entanglement to strategies. However, he shows that the odds of winning if both players use quantum strategies are the same as in their classical form.

IV. Conclusion

The implementation of classical quantum algorithms [57] in game theory does not seem to stand solely on the computational techniques used. Quite the contrary, there is a broad debate about changes in strategic models and payment results. Furthermore, in a real decision-making scenario, the intervention of the human being discredits the formulated mathematical formality. Consequently, the predictions of quantum computers seem to be closer to real life scenarios.

It should be considered that the study of the quantum implementation of classical game theory only takes two decades of research. With few published works if we compare them with other disciplines. Therefore, debate and interpretation are still open to great scientific contributions.

Future work will comprehensively address the implementation of binding cooperative and mixed strategy quantum games that include combinatorics.

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