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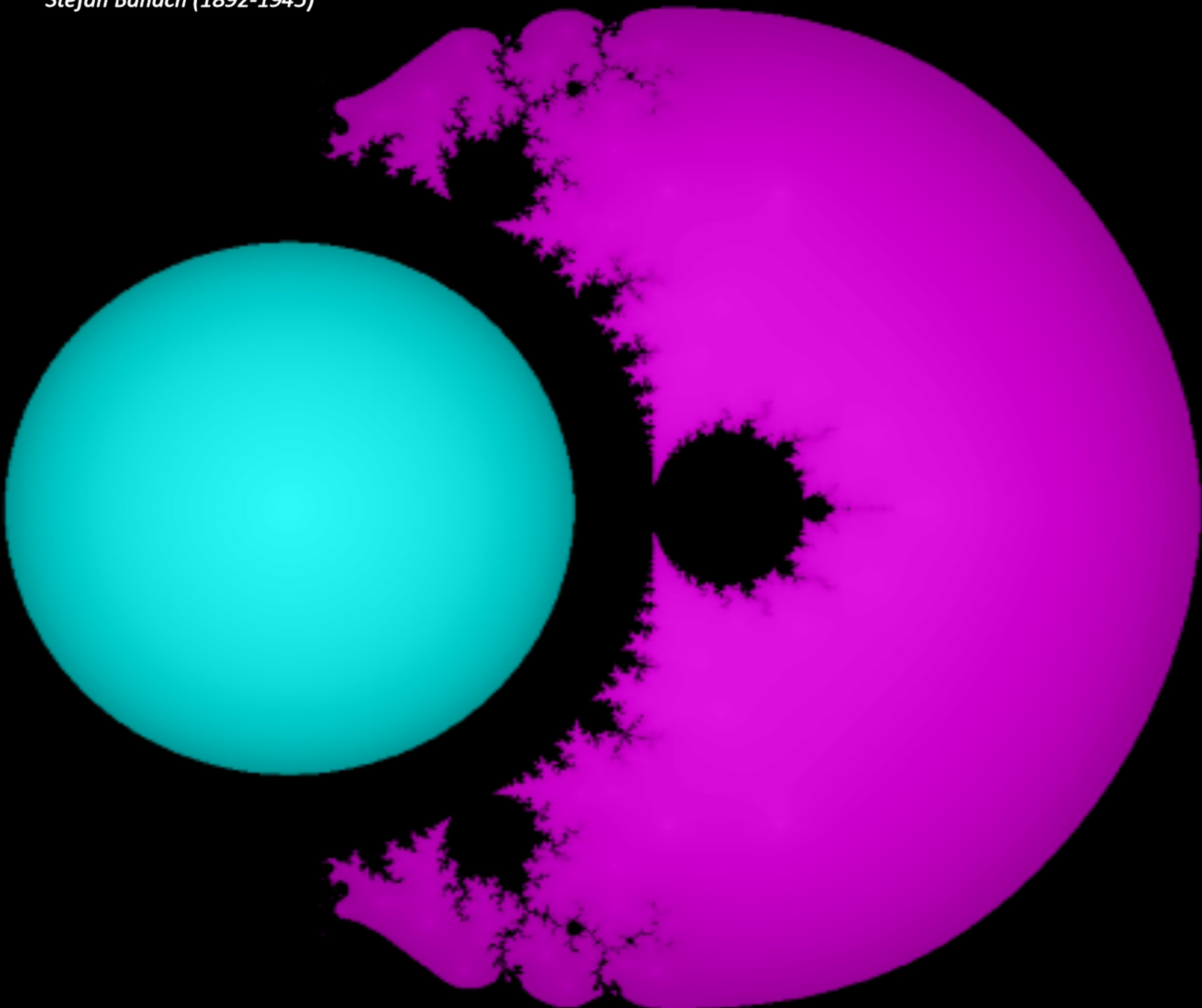
Mathematics is the most beautiful and most powerful creation of the human spirit.
Stefan Banach (1892-1945)



Special Issue on Teaching Mathematics Using New and Classic Tools

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Editor's Note

The International Journal of Interactive Multimedia and Artificial Intelligence provides an interdisciplinary forum in which scientists and professionals can share their research results and report new advances on Artificial Intelligence and Interactive Multimedia techniques.

This special issue, Teaching Mathematics using new and classic tools, concentrates on the practical and experimental teaching in advanced Mathematics in Engineering. The selected papers deal with the most relevant issues in the field, such as Mathematical learning and other different subjects in which Mathematics are needed, Advanced Mathematics, the development of different studied using Masive Open Online courser (MOOCs) or even the history of E-Learning and Mathematics. The result is a collection of experimental validations, mathematical papers and MOOCs studies which constitutes a clear contribution to the state of the art.

Teaching Mathematics is always hard since most of the students have the feeling that this field is not really applicable and not interesting at all. Although it is well known that Mathematics are present in almost all actions of life, as for exampe the design of a car or even shopping, it is complicated to make people understand this matter due to the fact that this Mathematics are camouflaged in the background. Moroever, there exists another problem: Student find Mathematics as a very tough subject, which is comupulsory, so many of the students have no motivation at all with the subject which constitutes another worry to teachers.

On the other hand, it is also known that one of the main benefits of using ICTs in a classroom is that teachers can motivate their students in almost every area and taking into account that, as previously remarked, this area is specially hard, we have to investigate all the possibilities that can make teaching easier. In this scene, E-Learning can be a very useful tool and we, as teachers, must get all the juice to it. The use of ICT and E-Learning have been increased exponentially in recent years both in school/college classrooms and University classrooms up to flood them with resources for a more meaningful learning in all areas of knowledgement.

In the last years, many authors are concerned with this problem and are putting all efforts in desgining new Websites related to Mathematics and their teaching (see [1]), investigating how math skills are acquired in different virtual environments (see [2]) or even developing different E-Learning environments which allow teachers evaluate their students in a compact and computerized way (see [3]). Therefore, many other authors have focused their studies in the field of teaching mathematics using E-Learning (see for example [3], [4] and [5]).

Over the past decade, the number of students that need specific mathematical courses is increasing every year leaded up to the requirement of developing different MOOCs which help students to understand better the basic concepts and can allow them to see different practical examples. Some authors have study this Mathematical MOOCS (see for example, [7], [8] and [9]).

Finally, the increasing number of research mathematicians has experienced an extraordinary growth and all the studies indicate that this trend will continue in the coming years. Some of this recent research is focused on developing new computer tools (see [10]) that can be used both as an aid to the investigation as in the classroom in order to explain advanced Mathematics.

Dr. Ángel Alberto Magreñán Ruiz

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Maths: from distance to e-learning

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***Abstract* — New technological progress and especially the use of Internet have implied a new paradigm on education, and nowadays one of its most prominent features is the rise of a new approach based on an instruction beyond the solid walls of schools and characterized by mobility. That is, e-learning. However, its origins and concept can be traced in time. This paper, focused on mathematics, deals with its evolution, antecedents and present status.**

***Keywords* — e-learning and b-learning, History distance learning, Maths competencies and Maths software and platforms.**

I. Introduction

Virtual Education: A new learning concept.

Louis Hector Berlioz said once: "Time is a great teacher, but unfortunately it kills all its pupils". How do we understand urgency? Distance? Nowadays' need of reach? Distance education is not contemporary, not even new. This type of learning in which pupils do not need to go to in-person classrooms or schools has been a fact since almost the very first education systems in mankind history. However, nowadays, there is something else new. Time, so respected by the French composer from the 19th century, has become more valuable if possible. Moreover, time is asked to be more flexible in its daily use. Similarly, the idea of distance has changed. The Euclidean space that equals to the length of a straight line has lost its *raison d'être*. In our society distance education lies in a virtual space through E-Learning. It has generated new values to measure time-space: the already mentioned time flexibility, globalization in the reach of education, costs decreases or fast teacher-student feedback…

In that new education context, it might be Urania who, jealous of Clio, wanted to stop being an exact science to widen its didactic understanding boundaries and, like History, make mathematics more versatile within a new educational paradigm. As Fernández-Mayoralas mentions, Gödel's theorems or fractal geometry blurred, lots of decades ago, the division between "sciences" and "humanities" [1 pp. 275-309]. However, in the time being, Mathematics might be using them and the humanistic conception of teaching to fit them again into the virtual educational world. Diatribes aside, according to Quinn, we should understand E-Learning as a "macro concept" with a short but intense history in terms of chronology. As that author said, "E-Learning is the macro concept that includes the mobile and online learning environments" [2]. Therefore, this new pedagogical mosaic consists of dozens of conceptual tiles that perform individually, but who all together form a renovating whole with its own contrasts.

The origin of that history of virtual change and transformation in education occurred outside networks and computers. As it has already been mentioned, in places where teacher-pupil distances were big, communication always looked for an artful way to develop itself. Thus, for instance, in the Australian Outback -also known as the "never never" desert- lessons were given through radio, from the small Alice Springs to the farms that were hundreds of kilometers far away. Besides, teachers visited their pupils by small airplanes once or twice a year [3 pp. 126-128].

After the first generation of distance teaching that would turn into paper, radio or television, new and fast evolution and perfection models would arise. The already mentioned first generation was developed between 1890 and the early 1970s. Nevertheless, following generations were very quickly established [4 pp. 217-219]. Thus, straddling those generations, we find Craft-Learning. Such system is based on teaching and training processes according to face-to-face classroom education with different communications through audio or videoconferences. As we can see, there have already been included elements from the second generation: video, tapes... That is the well-known "multimedia learning" [5 p. 57]. As far back as 1989-1999, the Uruguayan Edison Enrique Reina designed a new Home Learning (H-Learning) model with an IBM S/36 System. That method is still used and includes two activity sections: in-home lessons (school hours at home) and working and tutoring section, either *in situ* or face-to-face (FTF) with a simultaneous virtual presence. This pattern creates "personal learning environments" and, at the same time, brings together partial attendance courses and virtual learning [6]. That system is clearly connected to the one that would arise from the third generation of distance education or "telematic education". This one consists of a mixed education system called Blended learning or B-Learning [7 pp. 71-72] [8 p. 82]. Between 1985 and 1995 -when internet started to become public- that education system tried to combine again face-to-face learning with an emerging, but still uncertain, online learning. Therefore, it introduced into the learning models not only audiovisual media but also software designed to develop online courses. Actually, the usual face-to-face classroom model moved to the student's remote location. Obviously, it existed certain educator-student interaction, but the format was still "teacher-centered" and provided little feedback [5 p. 58].

The deficiencies of the already mention first stages started to be fixed from the mid-1990s thanks to the development of the full virtual learning or E-Learning. In that moment, as a fourth generation, student-tutor or student-teacher feedback became constant. Its interaction reaches forums, e-mail, wiki environments, chats, blogs, etc. [5 p. 58]. Then, we can talk about internet education. A kind of education that did not get stuck in a -virtual- space that embraces plenty of activity and that, besides, continued evolving in the last decade towards the Web 2.0 [9]. However, the path it was taking went beyond that level and moved towards the "3.0 non-browser" environment. New terms such as semantic web, cloud or the visual-spatial concept of the virtual 3D world would end up surrounding E-Learning. Such progress allows us to talk about an education that is not only detached from the classroom but also from a specific geographical area. In other words, it is an "immersive education" [10]. The device has been "released" –wireless- from the static terminal network to become mobile. It is in that point when the fifth and last generation of distance education begins, thanks to Mobile Learning (ML) and U-Learning [11 p. 264] [12 pp. 95-96] [13 pp. 23-24]. Since 2005, these options have allowed flexible learning in terms of location and timing factors. They offer a more automatized and direct communication through different database systems or simulations in virtual reality spaces [5 p. 58]. Thus, if ML allows us to access our learning material through mobile phones or tablets, U-Learning opens the doors to educational activity areas in a "ubiquitous reality" with an even non-existent space [14] [15 pp. 125-127]. Moreover, it has allowed the development of new and very accessible teaching spaces within the OER -Open Educational

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Resources-, for instance, the OCW -Open Course Ware- or the MOOC -"Massive Open Online Course"- for Universities [16] [17] [18] [19] [20] [21].

At the conceptual level, education in virtual spaces -the E-Learning macro concept- should not be a mere bazaar of pedagogical terminology at the service of the user. Such reality should not be understood as a miscellaneous market with more or less new didactic techniques. A new epistemological idea arises before us, where knowledge must sail between the sea of the old truths and that of the new beliefs. We are offered a new Educational Paradigm, in capital letters. In it, every discipline -such as Mathematics- must have space and place to develop. It is not, or at least should not be, a confrontation between the inappropriately called "traditional education" and "the new teaching ways". In contrast, it should be understood as part of a historical and evolutionary continuum. It is true that, until now, a distinction between "virtual" and "distance" education has been made [22 p. 90] [4 p. 220]. Its evolution has adapted to a cultural moment and era. In such context of teaching in the "virtual space", the didactics of human and exact sciences should start to settle. That is a path that, as we will see, Urania, the muse of Mathematics in old times, has taken so far.

II. From analogical to digital educational technology

As with many other disciplines, the origins of knowledge and learning of mathematics are lost in the mists of time and in fact both must be linked to the very human nature. In this way, as Bernard Duvillié said human being can be qualified as an *Homo Mathematicus* given that mathematical lore is accumulative on human society from its very beginning as it's illustrated by evidence from prehistoric and ancient societies [23] [24, p. 38]. Regarding the Greek experience, the precise term of mathematics is μαθηματική and it's derived from the noun μάθη which means learning or knowledge, implying the intimate conscience and deep importance of this field in ancient thought. As Plato said arithmetic, geometry and algebra "make people more useful to themselves, and more wide awake; and again in measurements of things which have length, and breadth, and depth, they free us from that natural ignorance of all these things which is so ludicrous and disgraceful" [25]. In fact, Greek mathematical knowledge wasn't at all set apart from philosophy and although basic mathematical teachings were taught on the primary level of education, complex mathematics were studied in the highest intellectual centers of antiquity as Plato's Academia, Aristotle's Lyceum or the Library of Alexandria [26, pp. 13-178] [27] [28, pp. 28-147] which were functioning until Christianism closed them. Certainly, distance learning wasn't a concept assumed in the Ancient and Medieval periods. However there was in Antiquity a kind of distance teaching in epistolary as letters were in some cases "assumed for teaching or instruction" [29, p. 5] between educators in any known discipline and pupils or enthusiasts, or were just written like a simple approach to knowledge's dissemination [30]. In this way, the letters addressed to the physicist and philosopher Hipatia by her disciple the bishop and poet Synesius of Cirene in the fifth century A.D. are deeply stimulating [31].

The fragmentation derived by the end of the Western Roman Empire and the arrival of newcomers to the imperial soil, the role of Christianism in education and culture and several other factors meant a rupture on the ancient education system. Regarding distance learning, it's necessary to pass through a chronological gap of more than one thousand years and continue this narrative at the nineteenth century when the Industrial Revolution implied a new technological paradigm and especially on communications.

If we keep on mind two simple but suggestive definitions of King; Young; Drivere-Richmond & Schrader [32] of distance learning and distance education, it's possible to trace landmarks, experiences and ways of educational transmission through time and space. Concerning distance learning, these scholars defined it as "improved capabilities in knowledge and/or behaviors as a result of mediated experiences that are constrained by time and/or distance such that the learner does not share the same situation with what is being learned". On the other hand, "distance education is formalized instructional learning where the time/geographic situation constrains learning by not affording in-person contact between student and instructor". Regarding the former notion, these authors consider two categories of distance education: synchronous and asynchronous, depending on the interaction established between tutor and learner [33, p. 212]. There's no room on this paper to deal with every kind of formal and/or informal evidence of learning so the focus will be on just some of the most significant means of distance educative transmission but always insisting on the mediating role of the instruction. That is, a book can provide knowledge but not learning or at least an awareness of bidirectional learning involved between an educator and a student.

As it was mentioned above, the Industrial Revolution sets a milestone regarding communications. The improvement on land transports with the apparition of railroads and the application of internal combustion engine, steamboats, the invention of telegraph and the deep development of post services implied a new time where technology matched with political, economic and social changes of crucial relevance to the immediate future [34]. One of the most important consequences was the emergence of mass society focused on consumption both material and communicational and this new frame obviously got reflected on education [35]. In fact, distance education is an evident sign of this new model.

The development of better postal services implied the appearance of the first correspondence courses and the first experiences come from the United States, as the Illinois Wesleyan University in 1873, the Correspondence University (Ithaca, NY) and the University of Chicago in 1883 illustrate [36, pp. 7-8]. This is the foundation for enormous present distance universities as British Open University or Spanish UNED [37]. Mass media as radio and television have had an enormous impact since their implantation on society. A curious account from 1924 of a tremulous observer describes perfectly the unexpected consequences of the impact of radio in education: "is radio to become a chief arm of education? Will the classroom be abolished, and the child of the future stuffed with facts as he sits at home or even as he walks about the streets with his portable receiving set in his pocket?" [35, p. 54]. Such an apocalyptical view has been frequently announced whenever any new mass media is established in society and although the basic frame of education hasn't changed, the impact is obvious and has supposed an immense chance for education on contexts where custom education wasn't available, as the Australian experience on the use of radio mentioned above or similar experiences show. In fact, the Ebola recent crisis in Africa inflicted a fracture on the educative system of Sierra Leone and the government alongside UNICEF launched a national radio education programme intended to teach almost two million of children across the country [38]. Actually, radio is a remarkable method of teaching because of the apparent simplicity, cheap cost and technical accessibility [39, ch. 6] and it's been used massively on several countries unable to create a traditional educational network or to isolated areas in Africa, South America, Asia and other world territories [40]. In effect, radio has many advantages from the cognitive point of view as this media contributes to the development of imagination and explores oral comprehension [41] and, in spite of the predominance of digital ICT, it still remains as an useful instrument for learning [42]. In comparison with radio, TV educational programmes have developed more frequently on most advanced countries because of the comparatively higher cost [43] since the first experience happened in USA in 1934 and especially after the end of the Second World War as TV is one of the most prominent features of the later consumer society and welfare state [44, p. 30]. In spite of its well deserved ill fame, TV also can be a very fruitful resource for distance education at least if there is a conscious guide on its use [45].

In sum, the impact and the advantages of distance learning are obvious. In the sixties, studies illustrated that Instructional TV implied the same learning results between traditional and distance education and this trend has been reaffirmed in later times [46]. It's not out of the ordinary to see that the biggest university in the world with 2'7 millions of students is a Chinese distance university named the China Central Radio and TV University. However, the technological advances mean new possibilities of distance learning and although radio, TV, correspondence, educational cassette tapes, and other analogical media aren't today such an important way of education in the age of internet they have their present share of relevance in learning as it's been argued above. However, the digital era proposes worldwide new possibilities as the African Virtual University (AVU) show. This is a pan-African university created originally by the World Bank and placed on Kenya, based on the use of satellite communications [47] [48].

III. The origins of e-learning

The development of online training and therefore of learning mathematics through the system e-learning follows the development of internet. From the 60s of last century the computer started out as a new tool available to teachers to improve teaching. With the advent of the internet the educational paradigm changes the traditional vertices formed by the physical space. Any content transmitted through the professor and the books as tools [49] will begin to change.

Internet growing has been evolving, improving and creating new tools that have led to areas such as education to evolve in the same way coming to include the network as part of their basic tools. The periodization or the evolution of internet and teaching with the help of computer and network in general is a matter, which with shades, tend to be divided into four or five major periods [49] [50, pp. 214].

A. From the 70s to middle 80s

Until the widespread use of computers, teaching with computer was carried out as a discipline away from the others and in a physical room other than the usual and in which a specialized teacher taught management. This fact allowed to have rooms dedicated only to learning through this tool. In educational and labour levels there were complications such as having to move to a particular place to develop the teaching [4, p. 221]. The background of the e-learning can be found at this time where the learning contents were the last ties between teacher and student, and were called Computer-Based Training, (CBT), Web-Based Training, (WBT) [51, pp. 1] or commoly known as Computer Assisted Teaching (EAO) [52, p. 14].

B. Multimedia (Late 80s-Early 90s)

The appearance of the Windows computer systems for personal computers was a very important step forward the progress of teaching on the whole. This period is characterized by the marketing of the CD-ROM, which will gradually replace the 3-1/2 5-1/4 diskettes.. Its ability to provide anytime, anywhere information brought about a quantum leap for multimedia training. The former publishers began to design programs and digital books were sold to educational institutions. These elements called "ToolBooks" were used as multimedia examples, in which it was possible to find topics related to the mathematics, algorithmic simulations, graphs, videos or games [53]. These multimedia first worlds represented a qualitative leap forward in the management of media. This issue allowed the management of the information in only one means system like the image in diverse facets, the audio and the contents. This system was a step forward in the development of teaching contents and mainly on topics like mathematics [54, p. 81].

C. Emergence of e-learning (Decade 90s)

The development of computer languages and the evolution of the web pages, among other elements, led to the beginning of the creation of pages on the internet related to mathematical applications. Intranets began to be common instruments in many schools. This improvement was accompanied by the development of specific tools for teaching as were the e-mail, the audio and video, Java applications and HTML. One of the tools most commonly used in these years was the tutoring by e-mail that has reached our days. But the real step forward for the emergence of the e-learning were the first online courses. At the beginning, in a first time in most cases it was only a question of the conversion of different documents to formats like Word, Acrobat or HTML and later to hang them on the web [55, pp.16-17]. Although the solution was not too original it served to give the first step towards the current e-learning, turning these static pages into dynamics offering a more attractive visual aspect, as it was the creation in Fern Universität of the course on line: "Mathematics for Economists" in which the improvements were made as interactive graphics, audio, video and animation, through Flash and Java [53, p.17] [56, pp 8-9]. The natural evolution of the on-line courses completed with the emergence of virtual courses. The forums in which course participants discussed, asked and presented their work, provided a dynamic constructivist bringing the physical classroom to the virtual scope [49]. Next to the forum, email, and the transfer of files became the basis of the current e-learning training. The challenge of this set of learning systems that will be framed within what we call as e-learning will be characterized by the communication through multiple channels, virtuality, the interactivity or the participation among other features [57, p. 32].

D. The consolidation of e-learning in XXI century

From models, early XXI century the e-learning has become the model to be developed, based on one in which cognitive factors mature virtual education in which cognitive and metacognitive factors that lacked it, began to consolidate. This is institutionalized through the famous Lisbon Strategy led by the European Council in 2000.

It assumes the need to understand the economic development through science and education as a policy to jointly develop [58, p. 10]. The final step is consolidated with the creation and development of tools and applications called Web 2.0, a term coined by Dale Dougherty and Craig Cline by 2004 [69, p. 367].It entailed the emergence of utilities and tools that the user was able to create their own content [60] [61, p.188]. Users spent "of being passive agents recipients of information to active agents capable of interaction, production and personal involvement in a new level of cooperation and interactive collaboration"[59, p. 368].The progress has led us to multiple variants of the teaching of mathematics e-learning. To understand the current situation of multiple existing systems within the e-learning including mathematics teaching García Aretio synthesizes this way (see table 1) [62].

TABLE 1
DIFFERENT E-LERNING SYSTEMS INCLUDING MATHEMATICS TEACHING

Synchronous mode	Asynchronous mode	Integrative mode
Courses to simulate the direct face interaction with teachers and classmates. Live video, conversations between teachers and students (chat) and these both together and voice conversations, online assessments, whiteboards where everyone can write or request clarifications.	The student decides the pace for learning and The availability of time. Student autonomy grows in this model. [50, p. 214]. We have low resolution video, evaluations offline, stored materials that students can handle at their personal convenience. Comunicación by e-mail and forums are usual in this model.	It Uses tools of the two previous models. [63] **First generation**: Adapts the existing classroom courses are adapted in the institution and the two systems are brought together. **Second generation**: The courses are created ad hoc and learning outcomes are equivalent to those taught in classroom format. **Third generation**: The courses are designed to the needs and training in the existing society.

Today there are other emerging models already quoted at the begining of the article but they are still unsophisticated such as the M-Learning, which refers to the use of mobile devices such as PDA, mobile or Windows CE for learning anytime and anywhere [62, p. 426] [64] [65]. Another model is known as Blended-Learning and one of the simplest and most accurate definition describes it as: "Which combines face-to-face and virtual teaching" [66, p. 12] [62, p.418]. It would be a review of the first mixed models based on semi-face contact between teacher and student [67] [68] [69, p. 18] [70].

IV. College and e-learning

The evolution of e-learning has taken us to the actual situation in which e-learning is a combination of integrated methods under a model or platform, rather than a homogeneous method. Each entity or institution uses it on a certain way, as you can see in higher education, where online teaching and e-learning are now an essential element. Colleges, although late, started this slow but inexorable process to develop and add this type of teaching. In 2008 a project which resulted in the creation of a White Book of Digital University began. One of the agreements that were pointed out was the responsibility of the Spanish colleges of having an electronic administration at the beginning of 2010 [68].

Since then until recently the e-learning outlook at college has made a qualitative leap forward.

If we follow the data provided by the Ministry of Education (2013) [71]., today the Spanish College System (SCS) is made up by 82 colleges in total, distributed in 348 campus, of those 112 belong to non on-site and special. Of these 82, 50 are public and 32 are private.

Since 2001, 14 private colleges have been created, four of them are non on-site colleges, making a total of 6 colleges of this type. These colleges are: Universidad a Distancia de Madrid, Universidad Internacional de la Rioja, Universidad Internacional Valenciana, Universidad Oberta de Catalunya, Universidad Internacional Isabel I de Castilla and Universidad Nacional de Educación a Distancia. All of them offer bachelor's degree and master, although masters are more demanded in general.

On Figure 1 the variable on enrolment is shown in % for official masters of campus universities and those that use e-learning during the last academic years.

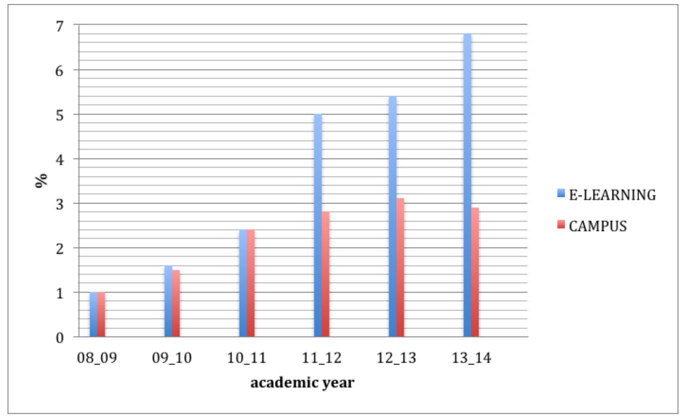

Fig. 1. Variable on enrolment in %

These results show how the increase in enrolment in e-learning colleges is larger than in campus universities, which is even stalled in the last period.

In the same way if we separate this enrolment between bachelor's degrees and masters, not only an increase is shown in masters enrolment, as it was supposed at the beginning, but also the increase in bachelor's degrees enrolment is clear.

This tendency is observed in Figure 2, where the enrolments in the Universidad Internacional de la Rioja and the Universitat Oberta de Catalunya are analyzed, used as examples because they are two of the private universities that are used as a model because they do all the teaching based on e-learning.

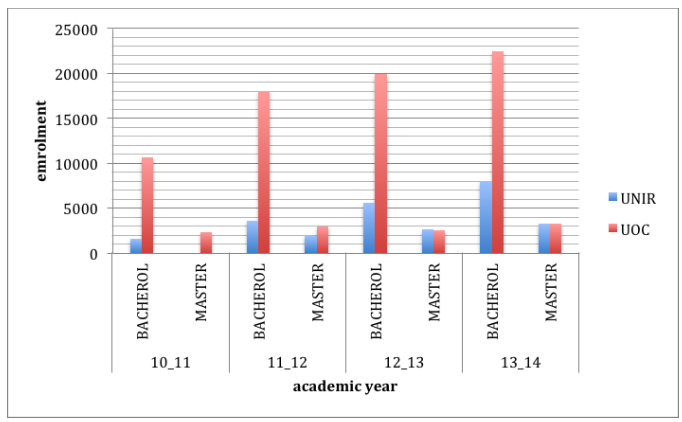

Fig. 2. Increase on enrolment in e-learning method

Enrolments in these colleges are centered in bachelor's degrees and masters in Social Studies, although gradually they are including bachelor's degrees and masters where science and technology are more prominent.

Also, it is interesting to emphasize that in high school in the last years the number of students enrolled that use this method of learning is also increasing, reaching 55,000 enrolled students in high school in 2014 and more than 1,500 in CIDEAD program, numbers that are increasing year by year.

V. Maths Competencies

Although the introduction of competences in the school system is relatively new, if we examine what math competence is, besides being a complex term and hard to define [72], we will see that it has substantially changed along the years, because it is no longer associated with the mechanical practice of some skills and routines. That is to say, not only is required the rote learning of the discipline and control algorithms, formulas, and procedures [73].

Although the authors and institutions that have tried to identify and enclose the term and concept of" math competence" are numerous [74] [75] [76], the most relevant author defines it as "the capacity to understand, judge, do and use math in a variety of contexts within mathematics and extra mathematics [75, p. 22] .

This author [75] goes further than this simple definition and categorizes it establishing two large groups of capacities or mathematical skills that have to take place in this competence: skills to establish questions and answer mathematical questions and with mathematics, and skills to use mathematical language and mathematical tools.

Based on the grouping we created before we have created the following table (table 2), where the categories that could be established on each group are shown [72].

TABLE: 2
GROUPS OF CAPACITIES OR MATHEMATICAL SKILLS

Capacity of formulate and answer mathematical questions and with mathematics	Capacity of manage mathematical tools and language
Think mathematically	Represent mathematical objects and situations
Set out and solve mathematical problems	Use mathematical symbols and formalities
Model mathematically	Communicate with mathematics and about mathematics
Reason mathematically	Use mathematical tools and resources

The OCDE [77], on the PISA report of 2012, to establish mathematical competence, uses Niss's definition [75] and also the categorization that he does of it, and points out that this capacity of the individual must include "the mathematical reasoning and the mathematical use of concepts, procedures, data, and tools that are needed to describe, explain and predict phenomena, as well as to recognize the role that mathematics have in society.

Although at the beginning up to eight mathematical capacities or skills are established [75], as we said before, the OCDE [77] regroups them in seven:

- Communication: in this skill, identify and understand the situation described in a problem is included, as well as the subsequent presentation of solutions.
- Mathematization: this term makes reference to the description of the main mathematical activities involved in a problem. What makes that a defined problem in the real world becomes into a strictly mathematical way.
- Representation: charts, tables, diagrams, images, equations, formulas, and specific materials.
- Reasoning and mathematical argument.
- Use of operations and symbolic, formal and technical language in a mathematical context.
- Strategy design to solve problems: selection or design of a plan to use mathematics to solve problems.
- Use of mathematical tools.

As it can be seen, this definition includes in an explicit way the use of mathematical tools, understanding as these the physical and digital equipments, computer programs, and calculation tools [77]. This has been created in order to have competent citizens that will use mathematical tools on e-learning regularly in their work environment as well as in their daily life [78].

VI. Virtual Learning environments, platform and math software

The introduction of competences in the Spanish educational system by the LOE [79] and after that, with the LOMCE [80], as well as the implementation of the European System of Higher Education in 2010 [81], have caused numerous methodological challenges in every educational level. If we focus on college education, specifically in higher education in mathematics and statistics as cross-curricular subjects in numerous bachelor's degrees and masters that are offered, we can observe that those challenges are mainly focused on the Internet implementation, specialized software, etc., and in the interest of the institutions to reinforce a diligent approach of these subjects, in order to add the professional competences linked to these knowledge areas [82]. Besides, the National Council of Teachers of Mathematics [83], emphasizes the importance of technology in the process of teaching and learning of mathematics and similar disciplines, and they include it in their standards.

After all the changes that the educational system faces it is necessary to design more effective learning environments and in agreement with actual demands [73]. It is in this context where the technologies of information and education arise as the main agent of change, due to their high amount of interdisciplinary nature, as well as the acquisition of the competence based on the technological, digital, and audiovisual literacy [84].

The use of these technological resources must not be reduced to just a modernization of the educational environment, they have to fulfill a specific function in the teaching-learning process. That is to say, the use of them must be ruled by an adequate and educational model, having an impact on methodology and in the use of technologies, not only in the knowledge of a technological tool. This means that we do not talk only about the ICT, but also these ICT that are effective in the teaching-learning process, that is to say, the technologies of learning and knowledge [85].

The introduction in the classrooms of the ICT and especially of Internet has created the learning virtual spaces, that are online platforms where the process of teaching-learning can be developed using the cooperative and / or collaborative learning [86]. It is necessary to say that not all the educational online platforms have to be considered as learning virtual spaces, because they not only have to encourage the student using different activities to build knowledge towards thinking, reasoning, and own reflection. Besides, the discussion and reflection with colleagues is also necessary [73]. In certain occasions this can be a problem, because sometimes this collaboration between students does not happen naturally, so the teacher has to be the one who encourages participation [87].

In the case of the disciplines related to the area of mathematics, besides the appearance of specific learning virtual spaces, and certain platforms, is joined by the widespread use of statistical and mathematical software that helps experimenting with concepts and techniques advanced, at the same time ensures a concept applied in this discipline, both in its scope, as transversely [88]. This software can be used both in virtual learning spaces, such as platforms, or just in isolation. Regardless of the form of use provides better visualization of mathematical and statistical concepts, an approach to mathematical-statistical knowledge by experimenting in different scenarios, the development of students' critical thinking, bridging the gap between theory and practice (allowing solving real problems), and of course represent a reduction of mechanical work [89].Therefore, although the students to achieve certain math skills is a complex task, yes these tools, facilitate, at first their achievement. Although keep in mind that to integrate digital technology, students must develop skills related to the use of ICT] [87]. In fact, he re-emphasized the PISA 2012 report [77] is specified that at the time of the assessment and not only are we considering math skills themselves, but also skills that cover aspects of mathematics and ICT, namely, that to master knowledge concerning needed to perform mathematical activities with the help of a technological device; and ICT skills.

VII. Platforms and specific mathematical software

A. Some nonspecific platforms

MOODLE:

https://moodle.org/?lang=es

It is a free distribution platform that allows you to create virtual learning environments through which to create and manage courses and create online learning communities. Numerous publications that test this platform as a virtual learning environment subjects of mathematics area [87] [90]. A priori, one of the problems that can make this platform is that your text editor does not allow the inclusion of mathematical symbols, however it is possible to integrate LaTeX filters that enables the interpretation of LaTeX code embedded in the text [90].

BLACKBOARD:

http://es.blackboard.com/sites/international/globalmaster/

Blackboard Academic Suite is a platform that enables the management and operation of courses for virtual learning environments. It is designed to facilitate and encourage the development of educational innovation.

SAKAI:

https://sakaiproject.org

It is a joint project of the University of Michigan, Indiana University, the Massachusetts Institute of Technology and Stanford University, the Open Knowledge Initiative (OKI) and uPortal consortium. It is based on the development of open source educational software to create virtual environments for collaborative learning in the field of higher education.

B. Specific platforms for mathematics

MUMIE:

https://www.mumie.net

It is the result of a joint project of cooperation between major European universities. It is an e-learning platform open to learning and teaching of mathematics and computer code. Both the courses and course materials are easily adjusted to any level of education. Have become virtual learning environments using social networks and wikis, to facilitate and encourage the creation of new content.

PROJECT DESCARTES

http://proyectodescartes.org/descartescms/index.php/latest/

It is a project developed by the Spanish government in the field of non-university education. Contains teaching materials for math, easily controlled by the teacher and simple to use by students. It is based on the contents of the official curriculum and allows the possibility of using active, creative and cooperative, while personal attention methodologies and, therefore, a good management attention to diversity.

In addition, the following table (table 3) lists specific mathematical software for different areas within the mathematical discipline, which can be used both in virtual learning environments independently as shown.

TABLE 3:
SPECIFIC MATHEMATICAL SOFTWARE

Area	Software name and URL addres
Computational Algebra	MÁXIMA http://maxima.sourceforge.net/es/
	SAGE http://www.sagemath.org/es/
	YACAS http://yacas.sourceforge.net/homepage.html
	MACULAY2 http://www.math.uiuc.edu/Macaulay2/
Numerical Analysis	GNU Octave https://www.gnu.org/software/octave/
	SCILAB http://www.scilab.org
Plotters	GNUPLOT http://www.gnuplot.info
	LABPLOT http://labplot.sourceforge.net
	SCIGRAPHICA http://scigraphica.sourceforge.net
Statistical Calculation	SPSS http://www-01.ibm.com/software/es/analytics/spss/
	R-PROYECT http://www.r-project.org

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ICTs and School Education

Nuria Aris and Lara Orcos

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***Abstract* — Nowadays, there exist lots of ICTs that teachers use as teaching tools. In this work, we introduce the theoretical context of the study of using ICTs in school education, then we present the method that will be used in order to achieve our goals. This work constitutes the groundwork to continue the study of ICT and its use in teaching.**

***Keywords* — ICTs, School Education, teaching paradigms.**

I. INTRODUCTION

We live in a society each day more digitalized and ICTs take part of both private and professional lives of most people. In the educational environment and student life cycle, interaction with ICTs begins in early childhood and lasts until the professional fulfillment. Technology has advanced and continues to progress, and with this dynamism its educational use is also modified. Therefore one of the main challenges of education is to make students know how to use technology as a tool for their own production, communication and learning throughout life.

In early childhood education we can find children very motivated by the use of ICTs-related games. This implies that, from very early stages, they are well acquainted with the use of phones, tablet and others. In a playful way, the child makes a progressive approach and that learning is in itself an initiation to ICTs.

At the stage of primary education, the child consolidates the instrumental learning and acquires basic skills. At this stage the constructivist pedagogical approach emphasizes the creation and production of knowledge by the student in relation to the environment. ICTs are used as teaching tools and resources to facilitate the development of the knowledge and skills of students. Therefore, it assumes great importance the role of teacher. Most teachers appreciate the benefits of ICTs because they represent a way to motivate students, to vary the classes, to individualize learning, etc. It is also true that the use of these technologies can cause them insecurity and rejection, since students typically handled ICTs better than themselves.

In this way, one of the fundamental goals of compulsory education is to prepare students to work in a digitized society.

In this article we reflect about it, about the impact of ICTs in education and the "how and how much" to use technology in the teaching and learning process. Equilibrium needs to be found as the use of ICTs should support the methods used to practice the competences required for a subject and the learning process of the individual.

II. METHOD

We propose a descriptive study, based on a deep review of specific literature, in order to show the relationship between the process of teaching and learning in ICTs, its theoretical foundation, application and regarding motivation, as well as the raise of the evaluation.

We intend to confirm the relationship between ICTs, its teaching process and the potential they represent in motivating students in the first, especially primary education stages. Our concerns are prompting us to give answers to questions such as:

- Is there a relationship between teaching and learning process with ICTs and motivation?
- What role do the teacher and the method used have?
- Can we establish relationship between playful motivation in ICTs and performance in a specific area?

Considering all these reflections, we initiated this first theoretical review as a starting point for contextualized in ICTs future research, based on leisure motivation and its application in instrumental learning and specifically in relation to mathematics.

III. THEORETICAL REVIEW

A. ICTs related contents

Before delving into the different dimensions presented above, it is interesting to establish a conceptual demarcation of the meanings associated with ICT terminology.

In this article 'the information and communication technologies' are used as a global term for all types of technique, including the computer, tablet, mobile phone, digital camera, interactive displays and the network. This term refers both to hardware, which can provide the components that are included on a computer system, and software, related to programs that can be used in the computer system and network resources. The term has an extensive content and includes both the treating information and the communicating concepts (Livingstone, 2012, Gulek et al., 2005).

In this study the sentence "A use of ICTs in education 'means that students use a laptop and resources it offers as well as those offered by the network. The focusing is that the students use it, not the teacher.

The phrase 'frequent use of ICTs' is used to describe the work of a group of students who used the computer and its resources in all educational activities in practice to develop all digital skills. All material used is in digital format, for example the training manual.

Comparing with the previous sentence, the phrase 'limited use of ICTs' is used in this study to describe the work of a group of students using computer and ICTs resources just to develop the digital competence. Computer and the resources it offers are considering as a complement to teaching and as a way to vary the usual teaching. The material used by students is usually printed. In the study, the frequent or limited computer use is approached from the perspective of the student, not from the teacher use.

In order to define 'digital skills' is used the definition of the European Communities (2012), which establishes that people should know how to use ICTs in a safe and critical way, both in their profession and in the spare time, and how to use them is used to communicate. The basic digital skills are "search, store, evaluate, produce, present and exchange information, and to communicate and participate in collaborative networks with the help of ICTs (European Commission 2012).

In the article the definition of the Royal Spanish Academy is used to describe the concept of 'education' as the 'action and effect of teaching "(RAE 2013). In the study, this is interpreted as the action of the teacher and what the student can do with the teacher's action.

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Another concept used, the concept of 'learning' is also defined with the help of RAE: "Action and effect of learning an art, trade or otherwise" (RAE 2013). This concept refers to the action of the student and the outcome of this process. It indicates that there is a permanent change in knowledge.

B. ICTs in School. ICTs evolution in the first stages of compulsory education

Technology has advanced and continues to progress, and it has also progress the way in which it is used. Today, the use is complex due to the fact that computers are connected to different networks and technology in general is more integrated into our lives. However, it is the connection between the use of ICTs and pedagogy is remarkable and this relation is in fashion at the moment.

Researching about it, four paradigms that represent different uses of ICT in education are found: The use of ICTs as an automatic device, as a tool, as a forum and as a means (Scheerens et al 2007).

These different paradigms of computer use exist in parallel, but were developed chronologically.

- The First paradigm, an automatic apparatus, is influenced by the ideas of behaviorism and structuralism (Valiente 2010). This paradigm is represented by educational programs in which students have to fill the hole left in a sentence with a word or a number. The student is active and receives direct feedback to his response. It is a mechanical work done by the student.
- The Second paradigm, the tool one, is influenced by the pedagogy of Jean Piaget. This pedagogy is based on the child's curiosity, the desire to know more and his own activity. The teacher's role changes from being someone who teaches another person to a person who guides students in their learning process. ICTs have supported this change of focus in the classroom. There are a high number of ways to use ICTs as a tool, such as finding written and recorded information or writing and making presentations. We can say that the purpose of teaching within this paradigm is that students use the computer for creative activities being much more active (Fleiser, 2011, Livingstone, 2012).
- The Third paradigm describes technology as an integrated part in our daily lives, as a forum for communication and interaction to share experiences and to play part in virtual worlds. There are many examples of this paradigm (Penuel, 2006).
- The last paradigm was added by Salinas (2005): the use of ICTs as a medium. In the school context, it means that school and administrative information is distributed through the network. Educational platforms are used and it also increase the number of distance courses that are conducted many times throughout the network. In this paradigm, the user is even more active. The development of the Internet has been a decisive factor for users since anyone has the chance to be published in the network. For school, it means that students can be published for example in a blog, and thus can reach a larger audience.

In the classroom today, it is common the mixture of methods based on different pedagogical tendencies due to the fact that the materials require different methods to afford different parts of subject and because students take advantage of a variety of teaching methods (Cabero et al 2005a and 2005b). The methods are not opposed in the teaching process, they complement.

What students do nowadays in classes is the same as it was done before, without digital tools (Vidal Puga 2011). Technology is used to find information, to write, and to support students to make presentations. There is no perceived a substantial difference between the use of ICTs in Spanish classes compared to other European countries (European Commission 2012), which means that the Spanish teachers, as European teachers, have remained cautious in regards to adopt entirely digitized class.

C. Constructivism

Teachers who use ICTs say that they do it because it is a way to motivate students, to vary power classes and to individualized learning. The authenticity of the material also attracts teachers to use technology. Often, the successful use of ICTs depends on the teacher. At the same time, there are teachers who express doubts about using ICTs as a lack of preparation in teaching applicability.

Thomas et al. (2013) suggest that contemporary ICTs play an important role in supporting the methods based on constructivism, adding an authentic teaching input, building relevant exercises for students and working cooperatively. It is important to remark that ICTs, themselves, are not a teaching method, however often facilitate or reinforce certain methods. So we can establish a direct connection between the development of ICTs and pedagogical ideas of constructivism (Vidal Puga 2006 Silvernail et al, 2011, Touron 2010, Slavin et al. 1996).

Indeed, there are those who say that the impact of ICTs on pupil´s performance does not appear without a constructivist teaching, that is, it's important to leave traditional methods away so as the use can be beneficial, which explains one of the goals the introduction of ICTs, to change the way we work in class more focused ways to the student. The "constructivism" is a pedagogical idea that emphasizes the creation and production of knowledge by the student in relation to the environment. In school debates, constructivism is often contrasted with 'tradition give lectures' by the teacher, in which knowledge is considered to be something transferable from one person to another. In this tradition, the teacher has a very pronounced role in class while in constructivism the student is focused in class. In the classroom today, it is common that both methods are mixed. In this situation, we consider the computer a tool and a resource in teaching that reinforces the teaching methods chosen.

This is evident in important aspects of constructivism such as motivation and autonomous learning. The student demonstrates the significant reflection process and the creation of increasingly complex and elaborate knowledge. This favors the creation of holistic knowledge (Pozo and Monereo, 2009, Touron 2010), contingent upon the competence learning. In this way the student, rather than acquiring knowledge, takes main role in its construction. It takes an active, autonomous and self-regulating role, since he takes the learning control. Throughout the ICTs, there are activated a high number of basic cognitive processes, including the selection and retention of information, the organization and development of new information, the integration of the information into prior knowledge and applying to the new knowledge situations (Huber, 2008). All this reverses in the student an increasing motivation for the task to developing (Peter, 2012).

The autonomous dynamic that sets the student himself is one of the most important and motivational aspects because he is the one who decides and takes leadership. Nevertheless, all this process depends on how the activity has been approached by the teacher. The teacher should be a helper and a guide on intellectual access to learning with and from ICTs. He should also focus the subject considering the perspective of the students: how they could better approach it, with what kind of difficulties they may come up with, what kind of media or additional support could be helpful, etc. (Zabalza, 2002).

There are studies that suggest that autonomy encourages them and they can achieve better results (Bolivar, 2007) .These elements can be the key in boosting the motivation for the learning and teaching process orientated the use of ICTs (Comer, 1999; Slavin, 1996) .

Taking into account the complementarities of the aspects described above, the proposed model is established, which articulates the teaching and learning process and with motivation and self-employment, all it aimed at achieving competences in the first educational levels.

In relation to these aspects it makes more sense to apply the concept of shared evaluation, which is defined as that evaluation process which is carried out jointly by the teacher and the student and also by the students considering the dialogue between the teacher with their students on the evaluation of learning and the processes taking place.

Thus, the act of evaluating is a joint construction between both parties. That is, it is an assessment from the interaction between the students themselves, with the teacher, and also from individual reflection itself.

We describe and explain these forms of assessment which should include the following levels simultaneously:

- Evaluation of independent work: Making own decisions is encouraged from the autonomous initiative. The student becomes aware of what its own learning process has brought him, arguing the "whys" and the perception of the degree of effort. It is interesting to mention that the self evaluation is not an automatic skill, is not a process that usually takes place, so it needs to be taught and supported (De Miguel, 2004 and 2006). It is important the role of the teacher in this learning guide (Alvarez et al. 2007). Knowing how to self evaluate is inextricably linked to the skills of critical thinking and the achievement of basic skills.
- The joint evaluation: The joint evaluation is a constitutive element of formative assessment that has a dual function, since it allows us to keep a record of the actions taken by students. It is based on the following levels of analysis:
- The first level, which can be described as "outside" made by both the teacher and the rest of the students from the contrast between the different contributions of each student.
- A second level, "inside" level that allows the student's self-assessment. For all this, we understand that this practice promotes the building of metacognitive capacity of the student, who is responsible for the regulation of his own learning (Pan, Pan, or Chang Lee, 2010).

IV. CONCLUSSIONS

- We want to insist on the proper initial character of this study because we are at the first implementation of this experience. This review allows us to understand the benefits of the adoption of ICTs and its relation to the motivational effects in the early stages of education.
- It might be assumed that investment in ICTs implementation projects in recent years have increased and changed the daily classroom use. But the truth is that it seems that there is no much difference in the work done by students and very often there is only a qualitative change. Students seek and find information in different ways; they find more information that before and present it in ways that were previously impossible.
- In this way, considering this initial research, we believe it is crucial to continue the research on the teacher educational intervention for future implementations of ICTs.
- Its potential is sensed not only in motivation but in the degree of the learning level in the students. They value their own effort and their learning undertaken with ICTs. The mixture of freedom and advice as well as all the "scaffolding" which sets the use of ICTs and the sharing creates a certain cooperative learning that leads to obtain positive results.
- The peer assessment also shows us a suitable way to demonstrate the competence achievement. In this direction, Bain (2005) explains that the best results are achieved by those teachers who believe students can change.
- In this way we can gradually gain a greater understanding of the learning process of students and also a set of guidelines that can lead the intervention on the teaching and learning processes oriented to the basic skills achievement (Boekaerts, 2001). The results motivate us to keep researching and applying it.

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Unifying the classical approach with new technologies: An innovative proposal for teaching mathematics in engineering

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***Abstract* — The aim of this paper is to present a teaching experience developed in the Polytechnic University of Cartagena and, more specifically, in the subject of Mathematical Methods Applied to Civil Engineering, that belongs to the Master Degree of Paths, Channels and Ports. Our classes were a mix between the traditional system and the new educational system. Moreover, we tried to adapt the evaluation process to the new European Framework for Higher Education. We have used videos developed by us and by students in our classes. We have noticed that the interest and motivation in class has grown. Also the grades have improved. We did a survey during this academic year and the results were strongly positive for both students and teachers.**

***Keywords* — European Framework, higher education, new educational tool.**

I. Introduction

Media, and video in particular, are in a period of profound transition [1]. There are several reasons for this outstanding development of this technology, but three stand out. Technology has rendered many of the processes of media creation, distribution, and consumption faster and less costly than ever before. Public expectations about the availability of media have grown to the point that many people consume and freely exchange media each day in the course of their personal and professional life. New companies and enterprises regularly make use of videos in publicity in order to influence fashion or to introduce new products in the market. YouTube, by posting 13 hours of video every minute, is one of the most powerful tools used for this tasks. Wikipedia, that is about to make available videos in its entries, via the open-source codec Ogg [2], will be another big distributor of video contents. While a number of studies [3] have pioneered the progress in the field of understanding the use of the Internet and web resources in secondary education and everyday life, the time is right to take a careful look to the teaching and learning at the university level [4].

We describe our experience using videos to teach Mathematical Methods to students of master degree level at the Polytechnic University of Cartagena. This proposal could be classified halfway between the classical teaching techniques and the new pedagogical proposals introduced to adapt teaching methods to the new European Framework for Higher Education. The innovation is focused on the use of teaching videos developed by teachers themselves as a complement for classroom lessons. We try to study the effect of these videos on our students' grades.

In the paper, we attempt to describe our experience. First of all, we perform a description of the subject. Secondly, we present the type of evaluation followed and the grades obtained by students. After that, we show the teaching evaluation survey. Finally, we expose some conclusions.

II. A Brief Description Of The Subject

The course of Mathematical Methods applied to Civil Engineering is composed by a collection of lessons that are taught during the first semester of the first year of the Master Degree level of Paths, Channels and ports Engineering. The course has a total of 30 hours, divided in 24 hours of theory an 6 hours of computer practices. It is a continuation of the knowledge acquired by the students during the undergraduate period. The aim of the subject is to widen the mathematical knowledge and tools that the students already have, and that is necessary to tackle the study of the rest of the subjects of the master degree. This subject will help to form the scientific profile that will be needed to assure professional expertise in their field.

The different parts of the subject are the following:

1. Preliminaries: Complex variable, differential geometry, form operators, surface geometry.

2. Introduction to differential equations: First order differential equations. Linear equations and systems of equations. Numerical resolution of first order differential equations.

3. Partial differential equations: Introduction to partial differential equations. Linear partial differential equations of second order. Separation of variables to solve PDEs. Integral transforms to solve PDEs. Numerical methods for PDEs.

4. Statistics and optimization: Non linear optimization. Function approximation through linear and non linear mean squares.

Computer practices consist in the manipulation of MATLAB programs provided by the teachers of the subject. The blocks of this part are the following

1. Numerical resolution of differential equations.

2. Numerical resolution of partial differential equations.

3. Statistics and optimization.

The study of the subject is organized in the following way:

1. Preliminaries: The study of this introductory part is done by the students alone. In order to do this, they use the material and notes provided by the professors of the subject.

2. Introduction to differential equations and partial differential equations: This part of the subject is introduced using the classical big group class. Theory and problems are exposed using the blackboard and slides.

4. Statistics and optimization: This part of the subject is studied in groups of two or three people using the videos provided by the professors of the subject.

5. Computer practice: This part of the program is introduced during the classes at the computer lab.

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III. Type Of Evaluation And Grading

The evaluation was divided in four sections and was carried out through exams, where theoretical and practical questions are posed to students. The aim was to check the level acquired by the students and their capability to apply their theoretical knowledge to particular problems:

1. Preliminaries. This part was graded with a ten percent extra for the whole subject, that could be obtained in the first exam.

2. Introduction to differential equations: This part was graded with the thirty percent of the total.

3. Partial differential equations: This part was graded with the thirty percent of the total.

4. Statistics and optimization: This part was graded with the thirty percent of the total.

5. Matlab programming: This part was evaluated including section in each one of the previous exams. The aim was to evaluate the programming skills acquired by the students. The total grade for this part was the ten percent of the total.

The grades earned by the 16 students enrolled in 2007-2008 were:

- 1 First of the Class with Distinction (10 of 10).
- 9 A's (9 of 10)
- 13 B's (7 of 10
- 1 C (5 of 10)

Since the academic year 2005/2006, annual teaching-evaluation surveys have been carried out for all subjects at our university. Below we present the grades earned, on a scale from 0 to 5. We only include the academic year 2014-2015:

- Teacher average grade for this subject: 4.3
- Degree average: 3.5
- We can see that the average for the subject is nearly one point above the mean of the degree.

IV. Analysis And Conclusions

The teaching experience described is a hybrid between the classical teaching methods and innovative strategies, that are necessary to adapt the subjects to the new European Framework for Higher Education. The greatest innovation was the use of teaching videos for an important part of the subject and the opportunity to pass the subject preparing and exposing projects through workgroups. The results achieved by the students and reflected in the teaching evaluation of this subject have been very positive.

From this experience, we can highlight the following points:

- The exposed proposition is in continuous evolution. Every year we actualize and complete the database of videos. Next year we are planning to introduce virtual blackboards, in order to record the expositions of the students. We think that it can be very interesting to provide access to expositions of students performed in previous years.
- The combination of promoting self-teaching and the fact that students like new technologies, is a good explanation of why the proposition exposed has been so successful.
- We have promoted in a natural way working in groups and collaboration among students.
- The work done by students along the semester must be continuous. A direct consequence is that classes are more dynamic.
- The fact that students must prepare and explain to their fellows sections of the subject, impose them a deep analysis of the part that they have to expose. For this reason, we think that expositions in public are highly pedagogical. When students have to prepare an exposition in order to explain to others, they pay attention to details that would go unperceived if they only had to prepare an exam by solving exercises. What is more, they also have to express themselves in a correct mathematical way, that is something quite complicate for most of the students.
- Individual tutorial sessions to each group of students has proven to be very useful for both, the students and professors. On the one hand, we can detect points of weakness in their process of learning. On the other hand, we assure more control over the work the students are doing. These tutorial sessions in reduced groups. enhance the relation between students and instructors, allowing the students to fight against their usual lack of confidence and to ask everything they have not understood during classes. Also, these activities grant that the students will plan better their preparation to confront the evaluation.
- We think that the proposed structure of the subject is correct. Our philosophy is to tackle the teaching of the subject globally, but using different approaches. First of all, we explain each part of the subject. Then, the students must be capable of assimilating the matter making use of the mentioned explanations, videos provided, explanations of their fellows in public presentations and tutorial sessions.
- We have also started to introduce talks where people from different research groups of our university explain a topic related with the subject. These talks have a great acceptance by the students and we think that are the perfect complement to show particular applications of the field studied.
- Finally, we want to remark that our proposal provide the students with the time that is necessary to mature the ideas exposed. This is of great importance when teaching mathematics.

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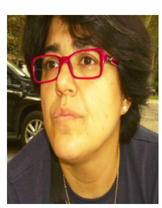

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Local convergence for an improved Jarratt-type method in Banach space

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Abstract **— We present a local convergence analysis for an improved Jarratt-type methods of order at least five to approximate a solution of a nonlinear equation in a Banach space setting. The convergence ball and error estimates are given using hypotheses up to the first Fréchet derivative in contrast to earlier studies using hypotheses up to the third Fréchet derivative. Numerical examples are also provided in this study, where the older hypotheses are not satisfied to solve equations but the new hypotheses are satisfied.**

Keywords **— Jarratt-type methods, Newton's method, Banach space, convergence ball, local convergence.**

I. Introduction

In this study we are concerned with the problem of approximating a solution x^* of the equation

$$F(x)=0 \tag{1}$$

where F is a Fréchet-differentiable operator defined on a convex subset D of a Banach space X with values in a Banach space Y.

Many problems in computational sciences and other disciplines can be brought in a form like (1) using mathematical modelling [11, 12, 28, 30]. Moreover, artificial intelligence and e-learning are topics of increasing interest in recent years. Other authors and people from various other areas of expertise can follow these techniques to serve a community of learners. The solutions of these equations can rarely be found in closed form. That is why most solution methods for these equations are iterative. The study about convergence matter of iterative procedures is usually based on two types: semi-local and local convergence analysis. The semi-local convergence matter is, based on the information around an initial point, to give conditions ensuring the convergence of the iterative procedure; while the local one is, based on the information around a solution, to find estimates of the radii of convergence balls. In particular, the practice of Numerical Functional Analysis for finding solution x^* of equation (1) is essentially connected to variants of Newton's method. This method converges quadratically to x^* if the initial guess is close enough to the solution. Iterative methods of convergence order higher than two such as Chebyshev-Halley-type methods [5, 6, 11, 12, 19-27, 29, 30, 32] require the evaluation of the second Fréchet-derivative, which is very expensive in general. However, there are integral equations, where the second Fréchet-derivative is diagonal by blocks and inexpensive or for quadratic quations the second Fréchet-derivative is constant. Moreover, in some applications involving stiff systems, high order methods are usefull. That is why in a unified way we study the local convergenve of the improved Jarratt-type method (IJTM) defined for each $n=0,1,2,\ldots$ by

$$\begin{aligned}
u_n &= x_n - F'(x_n)^{-1}F(x_n),\\
y_n &= x_n + \frac{2}{3}F(u_n - x_n),\\
J_n &= (6F'(y_n) - 2F'(x_n))^{-1}(3F'(y_n)+F'(x_n)),\\
z_n &= x_n - J_nF'(x_n)^{-1}F(x_n),\\
x_{n+1} &= z_n - (2J_n - I)F'(x_n)^{-1}F(z_n),
\end{aligned}$$

where x_0 is an initial point and I is the identity operator. If we set $H_n = F'(x_n)^{-1}(F'(y_n)-F'(x_n))$, then using some algebraic manipulation we obtain that

$$J_n = \frac{1}{2}(I + (I + \frac{3}{2}H_n)^{-1}) = I - \frac{3}{4}(I+\frac{3}{2}H_n)^{-1}H_n. \tag{2}$$

This method has been shown to be of convergence order between 5 and 6 [28, 32]. The usual conditions for the semilocal convergence of these methods are ($\mathcal{C}$):

- There exists $\Gamma_0 = F'(x_0)^{-1}$ and $\|\Gamma_0\| \le \beta$, $\beta > 0$;
- $\|\Gamma_0 F(x_0)\| \le \eta$, $\eta \ge 0$;
- $\|F''(x)\| \le \beta_1$ for each $x \in D$, $\beta_1 \ge 0$;
- $\|F'''(x)\| \le \beta_2$ for each $x \in D$, $\beta_2 \ge 0$

or

- $\|F'''(x_0)\| \le \overline{\beta}_2$ for each $x \in D$, $\overline{\beta}_2 \ge 0$ and some $x_0 \in D$;
- $\|F'''(x) - F'''(y)\| \le \beta_3 \|x-y\|$ for each $x, y \in D$
- $\|F'''(x) - F'''(y)\| \le \varphi(\|x-y\|)$ for each $x, y \in D$, where $\varphi: [0,+\infty) \to [0,+\infty)$ is a non-decreasing function.

The local convergence conditions are similar but x_0 is x^* in ($\mathcal{C}_1$) and ($\mathcal{C}_2$). There is a plethora of local and semilocal convergence results under the ($\mathcal{C}$) conditions [1-31]. These conditions restrict the applicability of these methods. That is why, in our study we assume the conditions ($\mathcal{A}$):

- $F: D \to Y$ is Fréchet-differentiable and there exists $x^* \in D$ such that $F(x^*)=0$ and $F'(x^*)^{-1} \in L(Y,X)$;
- $\|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \le L_0 \|x - x^*\|$ for each $x \in D$;
- $\|F'(x^*)^{-1}(F'(x) - F'(y))\| \le L\|x-y\|$ for each $x, y \in D$;

and

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- $\| F'(x^*)^{-1}F'(x) \| \leq K$ for each $x \in D$, $k > 0$.

Notice that the (A) conditions are weaker than the (C) conditions. Hence, the applicability of (IJTM) is expanded under the (A) conditions.

As a motivational example, let us define function f on $D = \overline{U}(1, \frac{3}{2})$ by

$$f(x) = \begin{cases} x^3 \ln x^2 + x^5 - x^4, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Choose $x^* = 1$. We have that

$$f'(x) = 3x^2 \ln x^2 + 5x^4 - 4x^3 + 2x^2,$$

$$f''(x) = 6x \ln x^2 + 20x^3 + 12x^2 + 10x$$

and

$$f'''(x) = 6 \ln x^2 + 60x^2 - 24x + 22.$$

Notice that $f'''(x)$ is bounded on D. That is condition (C_4) is not satisfied. Hence, the results depending on (C_4) cannot apply in this case. However, we have $f'(x^*) = 3$ and $f(1) = 0$. That is, conditions (A_1) is satisfied. Moreover, conditions (A_2) (A_3) are satisfied for $L_0 = L = 146.6629073\ldots$ and $K = 101.5578008\ldots$. Then, condition (A_4) is also satisfied. Hence, the results of our Theorem 2.1 that follows can apply to solve equation $f(x) = 0$ using IJTM. Hence, the applicability of IJTM is expanded under the conditions (A).

The paper is organized as follows: In Section 2 we present the local convergence of these methods. The numerical examples are given in the concluding Section 3.

In the rest of this study, $U(w, q)$ and $\overline{U}(w, q)$ stand, respectively, for the open and closed ball in X with center $w \in X$ and of radius $q > 0$.

II. Local convergence

In this section we present the local convergence of IJTM under the (A) conditions. It is convenient for the local convergence of IJTM to introduce some funcitons and parameters.

Let $L_0 > 0$, $L > 0$ and $K > 0$ be given constants. Define parameters r_A and r_0 by

$$r_A = \frac{2}{2L_0 + L} \tag{3}$$

and

$$r_0 = \frac{\sqrt{2}}{\sqrt{2}L_0 + L}. \tag{4}$$

Notice that

$$r_0 < r_A < \frac{1}{L_0}. \tag{5}$$

Define functions f_1 and f_2 on the interval $[0, \frac{1}{L_0})$ by

$$f_1(t) = \frac{L}{2(1 - L_0 t)} \tag{6}$$

and

$$f_2(t) = \frac{1}{3}\left(1 + \frac{Lt}{1 - L_0 t}\right). \tag{7}$$

Then, we have by the choice of r_A that

$$f_1(t) \leq 1 \quad foreach \quad t \in [0, r_A] \tag{8}$$

and

$$f_2(t) \leq 1 \quad foreach \quad t \in [0, r_A] \tag{9}$$

Define function f_3 on the interval $[0, \frac{1}{L_0})$ by

$$f_3(t) = \frac{(L)^2}{2(1 - L_0 t)^2}. \tag{10}$$

Then, we have that

$$f_3(t) \leq 1 \quad foreach \quad t \in [0, r_0] \tag{11}$$

and

$$f_3(t) < 1 \quad foreach \quad t \in [0, r_0) \tag{12}$$

Moreover, define functions f_4 and f_5 on the interval $[0, r_0)$ by

$$f_4(t) = \frac{Lt^2}{2(1 - L_0 t)}\left[1 + \frac{L^2 K t}{2(1 - L_0 t)^2 - L^2 t^2}\right] \tag{13}$$

and

$$f_5(t) = \left[1 + \frac{2K}{2(1 - L_0 t)^2 - L^2 t^2}\right] f_4(t). \tag{14}$$

Furthermore, define functions $\overline{f}_4$ and $\overline{f}_5$ on the interval $[0, r_0)$ by

$$\overline{f}_4(t) = f_4(t) - 1 \tag{15}$$

and

$$\overline{f}_5(t) = f_5(t) - 1 \tag{16}$$

We have that $\overline{f}_4(0) = \overline{f}_5(0) = -1 < 0$ and $\overline{f}_4(t) \to +\infty$, $\overline{f}_5(t) \to +\infty$ as $t \to r_0$. It follows by intermediate value theorem that functions $\overline{f}_4$ and $\overline{f}_5$ has zeros in $(0, r_0)$. Denote by r_4 and r_5 the minimal zeros of functions $\overline{f}_4$ and $\overline{f}_5$ on the interval $(0, r_0)$, respectively. Finally, define

$$r = \min\{r_4, r_5\} \tag{17}$$

Then, we have by the choice of r that

$$f_1(t) < 1, \tag{18}$$

$$f_2(t) < 1, \tag{19}$$

$$f_3(t) < 1, \tag{20}$$

$$f_4(t) < 1, \tag{21}$$

and

$$f_5(t) < 1 \quad foreach \quad t \in [0, r) \tag{22}$$

Next, we present the main local convergence for IJTM under the $(\mathcal{A})$ conditions.

Theorem 2.1 Suppose that the $(\mathcal{A})$ conditions and $\overline{U}(x^*, r) \subseteq D$, where r is given by (17). Then, sequence $\{x_n\}$ generated by IJTM for any $x_0 \in U(x^*, r)$ is well defined, remains in $U(x^*, r)$ for each $n = 0,1,2,\dots$ and converges to x^*. Moreover, the following estimates hold for each $n = 0,1,2,\dots$

$$\| x_{n+1} - x^* \| \le f_5(\| x_n - x^* \|) \| x_n - x^* \| < \| x_n - x^* \| < r, \tag{23}$$

where function f_5 is defined by (14).

Proof. We shall use induction to show that estimates (22) hold for each $n = 0,1,2,\dots$ Using $(\mathcal{A}_2)$ and the hypothesis $x_0 \in U(x^*, r)$, we have that

$$\| F'(x^*)^{-1}(F'(x_0) - F'(x^*)) \| \le L_0 \| x_0 - x^* \| < L_0 r < 1, \tag{24}$$

by the choice of . It follows from (24) and the Banach lemma on invertible operators that [11, 12, 27] $F'(x_0)^{-1} \in L(Y, X)$ and

$$\| F'(x_0)^{-1} F'(x^*) \| \le \frac{1}{1 - L_0 \| x_0 - x^* \|} < \frac{1}{1 - L_0 r}. \tag{25}$$

Using the first substep of IJTM for $n = 0$, $F(x^*) = 0$, $(\mathcal{A}_1)$, $(\mathcal{A}_2)$, (24) and the choice of r we get that

$$\begin{aligned} u_0 - x^* &= x_0 - x^* - F'(x_0)^{-1} F(x_0) \\ &= -(F'(x_0)^{-1} F'(x^*))[F'(x^*)^{-1} \\ &\times \int_0^1 (F'(x^* + \theta(x_0 - x^*)) - F'(x_0)\, d\theta (x_0 - x^*)], \end{aligned} \tag{26}$$

so

$$\begin{aligned} \| u_0 - x^* \| &\le \| F'(x_0)^{-1} F'(x^*) \| \\ &\times \| F'(x^*)^{-1} \int_0^1 (F'(x^* + \theta(x_0 - x^*)) - F'(x_0)) d\theta \| \| x_0 - x^* \| \\ &\le \frac{L_0 \| x_0 - x^* \|^2}{2(1 - L_0 \| x_0 - x^* \|)} \le \frac{L \| x_0 - x^* \|^2}{2(1 - L_0 \| x_0 - x^* \|)} \\ &\le f_1(r) \| x_0 - x^* \| < \| x_0 - x^* \| < r, \end{aligned} \tag{27}$$

which shows $u_0 \in U(x^*, r)$. Using the second substep of IJTM, we get by (27) and (19) that

$$\begin{aligned} y_0 - x^* &= x_0 - x^* + \frac{2}{3}(u_0 - x_0) \\ &= x_0 - x^* + \frac{2}{3}(u_0 - x^*) + \frac{2}{3}(x^* - x_0) \\ &= \frac{1}{3}(x_0 - x^*) + \frac{2}{3}(u_0 - x^*) \end{aligned}$$

so,

$$\| y_0 - x^* \| \le \frac{1}{3} \| x_0 - x^* \| + \frac{2}{3} \| u_0 - x^* \| \le f_2(r) \| x_0 - x^* \| < r,$$

which shows that $y_0 \in U(x^*, r)$.

Next, we shall find upper bounds on $\| H_0 \|$ and $\| J_0 \|$. Using $(\mathcal{A}_1)$, (26), (20) that

$$\begin{aligned} \frac{3}{2} \| H_0 \| &\le \frac{3}{2} \| F'(x_0)^{-1} F'(x^*) \| \| F'(x^*)^{-1}(F'(y_0) - F'(x_0)) \| \\ &\le \frac{3}{2} \frac{L \| y_0 - x_0 \|}{1 - L_0 \| x_0 - x^* \|} \le \frac{3}{2} \cdot \frac{2}{3} \frac{L \| u_0 - x_0 \|}{1 - L_0 \| x_0 - x^* \|} \\ &\le \frac{L^2 \| x_0 - x^* \|^2}{2(1 - L_0 \| x_0 - x^* \|^2)} < \left(\frac{Lr}{\sqrt{2}(1 - L_0 r)}\right)^2 \\ &= (f_3(r))^2 < 1 \end{aligned} \tag{28}$$

It follows from (27) and the Banach lemma on invertible operators that $(I+\frac{3}{2}H_0)^{-1}\in L(Y,X)$ and

$$\left\|(I+\frac{3}{2}H_0)^{-1}\right\|\leq\frac{1}{1-\frac{L^2\|x_0-x^*\|^2}{2(1-L_0\|x_0-x^*\|)^2}}$$

$$<\frac{1}{1-\frac{L^2r^2}{2(1-L_0r)^2}}. \tag{29}$$

It then follows from the definition of J_0, (28) and (29) that

$$\|J_0\|\leq 1+\frac{3}{4}\frac{\frac{L^2\|x_0-x^*\|^2}{3(1-L_0\|x_0-x^*\|)}}{1-\frac{L^2\|x_0-x^*\|^2}{2(1-L_0\|x_0-x^*\|)^2}}$$

$$=1+\frac{1}{2}\cdot\frac{(1-L_0\|x_0-x^*\|)L^2\|x_0-x^*\|^2}{[2(1-L_0\|x_0-x^*\|)^2-L^2\|x_0-x^*\|^2]}. \tag{30}$$

Then, from the fourth substep of IJTM for $n=0$, (27), (28), (29), (21) and (A_4)

$$z_0=x_0-F'(x_0)^{-1}F(x_0)+\frac{3}{4}(I+\frac{3}{2}H_0)^{-1}H_0F'(x_0)^{-1}F(x_0)$$

so,

$$\|z_0-x^*\|\leq\|x_0-x^*-F'(x_0)^{-1}F(x_0)\|$$

$$+\frac{3}{4}\left\|(I+\frac{3}{2}H_0)^{-1}\right\|\|H_0\|\|F'(x_0)^{-1}F'(x^*)\|$$

$$\times\left\|F'(x^*)^{-1}\int_0^1F'(x^*+\theta(x_0-x^*))(x_0-x^*)d\theta\right\|$$

$$\leq\frac{L\|x_0-x^*\|^2}{2(1-L_0\|x_0-x^*\|)}+\frac{3}{4}\frac{1}{1-\frac{L^2\|x_0-x^*\|}{2(1-L_0\|x_0-x^*\|)}}$$

$$=\frac{2}{3}\frac{L^2\|x_0-x^*\|}{2(1-L_0\|x_0-x^*\|)^2}\frac{K\|x_0-x^*\|}{1-L_0\|x_0-x^*\|)}$$

$$=f_4(\|x_0-x^*\|)\|x_0-x^*\|<\|x_0-x^*\|<r, \tag{31}$$

which shows $z_0\in U(x^*,r)$.

Notice that we used

$$F(x_0)=F(x_0)-F(x^*)=\int_0^1F'(x^*+\theta(x_0-x^*))(x_0-x^*)d\theta$$

so

$$\|F'(x^*)^{-1}F(x_0)\|\leq K\|x_0-x^*\|\quad by\quad (A_4). \tag{32}$$

Next, using the last substep in IJTM for $n=0$, (25), (29), (21) and (32) (for x_0 replaced by z_0) we get in turn that

$$\|x_1-x^*\|\leq\|z_0-x^*\|+\frac{1}{1-\frac{L^2\|x_0-x^*\|^2}{2(1-L_0\|x_0-x^*\|)^2}}\frac{K\|z_0-x^*\|}{1-L_0\|x_0-x^*\|}$$

$$\times[1+\frac{2K(1-L_0\|x_0-x^*\|)}{2(1-L_0\|x_0-x^*\|)^2-L^2\|x_0-x^*\|^2}]\|z_0-x^*\|$$

$$\leq f_5(\|x_0-x^*\|)\|x_0-x^*\|\leq f_5(r)\|x_0-x^*\|$$

$$<\|x_0-x^*\|, \tag{33}$$

which shows (23) for $n=0$.

To complete the induction, simple replace in all preceding estimates x_0,u_0,y_0,z_0,x_1 by x_k,u_k,y_k,z_k,x_{k+1}, respectively to arrive at (23), which complete the induction.

Finally it follows from (23) that $\lim_{k\to\infty}x_k=x^*$ ▪

Remark 2.2

1. Condition (A_2) can be dropped, since this condition follows from (A_3). Notice, however that

$$L_0\leq L \tag{34}$$

holds in general and $\frac{L}{L_0}$ can be arbitrarily large [2-6].

2. In view of condition (A_2) and the estimate

$$\|F'(x^*)^{-1}F'(x)\|=\|F'(x^*)^{-1}[F'(x)-F'(x^*)]+I\|$$

$$\leq 1+\|F'(x^*)^{-1}(F'(x)-F'(x^*))\|$$

$$\leq 1+L_0\|x-x^*\|,$$

condition (A_4) can be dropped and K can be replaced by

$$K(r)=1+L_0r. \tag{35}$$

3. It is worth noticing that r is such that

$$r<r_A\ \ for\ \alpha\neq 0 \tag{36}$$

The convergence ball of radius r_A was given by us in [2, 3, 5] for Newton's method under conditions (A_1)-(A_3). Estimate (24) shows that the convergence ball of higher than two IJTM methods is smaller than the convergence ball of the quadratically convergent Newton's method. The convergence ball given by Rheinboldt [30] for Newton's method is

$$r_R=\frac{2}{3L}<r_A \tag{37}$$

if $L_0 < L$ and $\frac{r_R}{r_A} \to \frac{1}{3}$ as $\frac{L_0}{L} \to 0$. Hence, we do not expect r to be larger than r_A no matter how we choose L_0, L and K. Finally note that if $\alpha = 0$, then IJTM reduces to Newton's method and $r = r_A$.

4. The local results can be used for projection methods such as Arnoldi's method, the generalized minimum residual method (GMREM), the generalized conjugate method (GCM) for combined Newton/finite projection methods and in connection to the mesh independence principle in order to develop the cheapest and most efficient mesh refinement strategy [11, 12, 30].

5. The results can also be used to solve equations where the operator F' satisfies the autonomous differential equation [11, 12, 28, 30]:

$$F'(x) = T(F(x)), \tag{38}$$

where T is a known continuous operator. Since $F'(x^*) = T(F(x^*)) = T(0)$, we can apply the results without actually knowing the solution x^*. Let as an example $F(x) = e^x - 1$. Then, we can choose $T(x) = x + 1$ and $x^* = 0$.

6. It is worth noticing that IJTM is not changing if we use the (A) instead of the (C) conditions. Moreover for the error bounds in practice we can use the computational order of convergence (COC) [1-4, 11, 12, 14] using

or the approximate computational order of convergence (ACOC)

$$\xi = \sup \frac{\ln\left(\frac{\| x_{n+2} - x_{n+1} \|}{\| x_{n+1} - x_n \|}\right)}{\ln\left(\frac{\| x_{n+1} - x_n \|}{\| x_n - x_{n-1} \|}\right)} \quad foreach \quad n = 1,2,\ldots$$

instead of the error bounds obtained in Theorem 2.1.

III. Numerical examples

$$\xi^* = \sup \frac{\ln\left(\frac{\| x_{n+2} - x^* \|}{\| x_{n+1} - x^* \|}\right)}{\ln\left(\frac{\| x_{n+1} - x^* \|}{\| x_n - x^* \|}\right)} \quad foreach \quad n = 0,1,2,\ldots$$

We present numerical examples where we compute the radii of the convergence balls.

Example 3.1 *Let* $X = Y = R^3$, $D = \overline{U}(0,1)$. *Define* F *on* D *for* $v = (x, y, z)$ *by*

$$F(v) = (e^x - 1, \frac{e-1}{2} y^2 + y, z). \tag{39}$$

Then, the Fréchet-derivative is given by

$$F'(v) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y+1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notice that $x^* = (0,0,0)$, $F'(x^*) = F'(x^*)^{-1} = diag\{1,1,1\}$, $L_0 = e - 1 < L = K = e$, $r_0 = 0.274695\ldots < r_A = 0.324967\ldots < 1/L_0 = 0.581977\ldots$, $r = 0.144926\ldots$.

Example 3.2 *Let* $X = Y = C([0,1])$, *the space of continuous functions defined on* $[0,1]$ *be and equipped with the max norm. Let* $D = \overline{U}(0,1)$. *Define function* F *on* D *by*

$$F(\varphi)(x) = \varphi(x) - 5\int_0^1 x\theta\varphi(\theta)^3 \, d\theta. \tag{40}$$

We have that

$$F'(\varphi(\xi))(x) = \xi(x) - 15\int_0^1 x\theta\varphi(\theta)^2 \xi(\theta) d\theta, \ foreach \ \xi \in D.$$

Then, we get that $x^* = 0$, $L_0 = 7.5$, $L = 15$ and $K = K(t) = 1 + 7.5t$, $r_0 = 0.055228\ldots < r_A = 0.066666\ldots < 1/L_0 = 0.133333\ldots, r = 0.0370972\ldots$.

Example 3.3 *Returning to the motivational example at the Introduction of this study, let the function* f *on* $D = \overline{U} = (1, \frac{3}{2})$ *defined by*

$$f(x) = \begin{cases} x^3 \ln x^2 + x^5 - x^4, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Then, $L_0 = L = 146.662907\ldots$, $K = 101.557800\ldots$, $r_0 = 0.003984\ldots < r_A = 0.004545\ldots < 1/L_0 = 0.006818\ldots$ and $r = 0.000442389\ldots$.

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Towards a better learning models through OCWs and MOOCs

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Universitat Politècnica de València

***Abstract* — Technological advances of XXth century have induced a profound change in society and, therefore, in the high education. Internet supposed a qualitative difference, as information and digital images flooded into homes around the world.**

The Universitat Politècnica de València (UPV) is a medium sized university of Spain that has been involved in the development of digital video content (Polimedia) to support teaching processes for several years. Joint with Polimedia and other learning objects (virtual laboratories, applets, etc.), the UPV promoted the construction of OCWs. Along with the improvement of technology, MOOCs appeared as e-learning material.

In this work, we analyze the advantages and drawbacks of OCWs and MOOCs when they are used in our classroom. This experience has led us to incorporate in our methodology the flip teaching.

***Keywords* — Digital video, Flip education, Learning objects, MOOC, OCW, Polimedia.**

I. Introduction

Since the late 90s, the growth of technology, particularly related to internet, media, generating videos, etc. has grown exponentially. Besides, this change has been accompanied by a clear cost reduction and eases of handling different devices for creating, editing, quality videos and then hang on internet almost automatically. The technology has taken over our homes substantially changing society in which we live.

For our students this change has not existed. When they were born, technology was already at their fingertips. They do not conceive a world in which to communicate one has to go to a device attached to a table or going to the library for references. They are also used to have a lot of information quickly and in quality formats that simplify the ideas they convey. We think that they have lost concentration and deepening skills into a topic. When they encounter any problem (whose solution is not accessible by Internet) that requires more dedication and time, they abandon for lack of patience, because the analysis of the problems and search of their solutions are activities that today's society do not encourage.

In the educational field, to the observed change in our students, teachers introduced new technologies in education firstly by themselves and, later on, with institutional support adapting the teaching methodologies.

The improvement of technological media has facilitated video manipulation and the uploading to Internet, has led naturally to the massive generation of didactical videos, being YouTube a clear proof of this.

In particular the Universitat Politècnica de València (UPV) decided in 2003 to develop Polimedia, a system to record HD video learning objects using cheap audiovisual studios in a fast and simple way [1] that will be distributed offline-through a Learning Management System, in our case Sakai [2]. For supporting the teacher's work UPV started a support program named "Docencia en red" (teaching network) that gives both technical and economic support to encourage teachers to develop digital learning contents and to assess its quality.

This program is still open today. OpenCourseWare (OCW) plays an important role among these materials. "These provide free educational content you can use to enhance or refresh your knowledge or teach others". They could be formed only by text or text including links, videos, applets, etc. (see [3]). The OCW constitute the first major step to allow free access to knowledge from anywhere in the world having internet, so as to facilitate learning on-line. This implies to publish all of our course materials online and make them widely available to everyone.

Moreover, Internet is mainly used as a means of interactive communication, giving voice to the user for input, collaborate, interact, etc., allowing these contributions to be returned to other users in the form of shared knowledge. This is what is commonly known as Open Social Learning.

The OCW and the Open Social Learning have been the basin, during the last decade, for the appearance of the first Massive Open Online Course (MOOC) in institutions of higher education. A MOOC is an online course with the option of free and open registration, a publicly shared curriculum and opened outcomes [4]. Since the term was coined in 2008 to name a connectivism course offered by the University of Manitoba, a lot of Universities have offered courses under this denomination.

However, OCW, MOOC and, in general, any online learning material, is not only useful for accessing training outside the classroom. In this paper we show our experience when incorporating them to traditional subjects, commenting on the advantages and drawbacks that we have found.

II. OCW and MOOC

The interaction between internet users with common interests has led to the generation of multiple manuals on specific topics. Easy access to these materials and their usefulness, has allowed many users to get visibility to their ideas and skills, which has led to an exponential growth of tutorials, videos, etc. that currently flood internet.

Education has not been immune to this technologic and social change, because we find today a wealth of information, either video or text, on how to solve all kind of problems, with different levels of complexity.

The increasing use of Open Social Learning is a clear evidence of the interest of internet users to meet their training needs online. Therefore educational institutions, where the appropriate technology was available, have conducted teaching projects in which a large number of students have been engaged. OCWs were the first organized teaching network.

The Open Education Consortium (OEC) [5] is a worldwide community of hundreds of higher education institutions and associated organizations committed to advancing open education and its impact

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on global education. The main goal of this consortium is to promote learning through the distribution and use of open free high-quality educational materials, organized into subjects, thus contributing to the free dissemination of knowledge.

Some Spanish universities, UPV among them, have created programs to encourage and support teachers that develop digital learning contents and to assess them regarding its quality. Among worldwide generated OCW subjects, in the first edition of the Awards of Excellence OEC (2011), the following subjects were awarded: "Actividad Fisica para la Salud" (Physical Activity for Health) taught by Pedro A. López-Miñarro at the University of Murcia, "Estructuras Matemáticas para la Informática II" (Mathematics for Computer Structures II) taught by the co-author of this work Cristina Jordan at the UPV and "Instituciones Básicas del Derecho Administrativo," (Basic Institutions of Administrative Law), taught by José Fernández at the University Carlos III of Madrid.

MOOCs are a new step in the evolution of this type of institutional online training [6]. Although their apparent similitude with OCWs, we would like to point out a significant difference: registration is required to access MOOCs because its goal is not only to present contents but also to carry out monitoring the student progress and being able to certify their use. Essentially these are packaged up pieces of learning that last a few weeks, are often put together by a top professor at a top university, and are available to anyone with a computer.

MOOCs are characterized by:

- Enrolment is unlimited
- Being online open
- Access and materials are completely free
- Teacher-student and student-student interaction are made via the forum, chat, e-mail, etc.

Currently, the certification of these courses works with the code of honor, but new ways for giving them an academic validity are under study. Among these new ideas being considered, we have those involving exams at partner institutions and also those using technological means to guarantee the authorship of the online tests during the course.

A first question is if the MOOCs are so great as they appear. Since 2012, the year in which they were known and used by the general public, there have been numerous statistics studies on its operation. Regarding the monitoring of these courses, only 10% of those enrolled finish them. On the other hand, they have not been useful to generalize higher education, since over 80% of those enrolled already had a higher qualification. The main problems are the lack of accessibility to Internet for students from poorer countries, the previous academic level or English language level, that is used in the most of courses [7].

A second question raised is if MOOCs with large-scale interactive participation and open access via web will replace face-to-face teaching in traditional universities, reducing the need of professors and universities. We believe that, by the comments of the previous paragraph, MOOCs will not pose a threat to classroom education, but on the contrary, they can be a useful tool in our classrooms. In the Fig. 1 we show the web of the MOOCs at UPV.

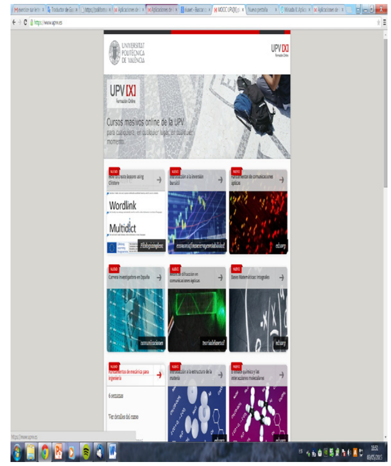

Fig.1. Screenshot of MOOCs of UPVX

III. Towards Flip Education

A. Starting situation

In college, although new technologies are used as a teaching support, an effort of comprehension and analysis is required. Just watching images is not enough to solve problems, it is necessary to adopt an active attitude. They should read and study texts, either on paper or file. It is hard to keep their attention, they easily get bored and mislead, and we observe that they need motivation to analyze any subject; they do not have patience to get results. In addition, they know that at any moment they can get immersed in the virtual world, getting away from what at that time seems boring and complicated. Moreover, working skill has also decreased; maybe because actual society takes care too much of them and tries to make their life too easy, or perhaps because they are used to think that things can be done with a click, or there is always a method to solve anything. So, to think for solving the problem is not necessary [8].

As a consequence, a common attitude observed during our years of teaching mathematics at a Computer Science and Engineering schools is the lack of interest in mathematics that, in general, show their students. Due to their lack of training in scientific understanding, they think that mathematical argumentations are hard to follow. In addition, they show difficulties in scientific expression, sometimes because the vocabulary is totally new. Finally, they sometimes show a poor level of motivation for the contents since they think that they are useless.

B. Some changes are possible

The previous considerations motivate us to conduct the following actions:

a) Introduce changes in the presentation of the contents of our subjects, showing real-life applications of the topics of the subject. This is done via modeling problems.

b) Modify the methodology in order to achieve the following goals:

- Make learning easier
- Students must detect their own mistakes or gaps
- Correct them in time
- Get, when they finish the course that they have learned beyond the concepts of the subject (which probably will be mostly forgotten, if they do not use them)
- Distinguish a logical reasoning from a wrong one
- Discover strategies for addressing to the problems more easily

- Improve their scientific understanding and their expression skills, very useful throughout their professional lives

C. How to carry changes out

Taking into account the big amount of accessible online material, we pose ourselves about what we bring and what we should bring in the classroom. Nowadays, we can access to the knowledge with a mouse click. If our class is limited to data exposure, we are wasting our time. In addition, new methodological trends indicate that students must participate in class, working individually or in groups, not only outside the classroom, but under teacher guidelines.

The problem of implementing these educational activities is often the lack of time. So, if we plan to modify our classroom sessions, it may be preferable that the first contact of the student with the contents of the course takes place outside of it, in an autonomous manner, appropriate to their level. For this, we must assign pre-class activities in order to get that time in the classroom can be used in more interactive tasks. So, classroom work will reach higher levels of Bloom's Taxonomy [9], as is the analysis, which requires further debate and discussion (see Fig. 2). The interaction with the teacher and other colleagues then becomes more necessary. Furthermore, this type of teaching allows a more personalized assessment of students.

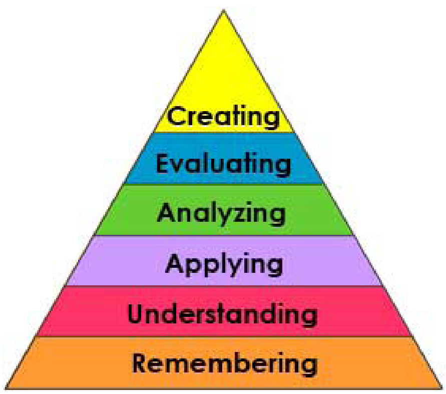

Fig.2. New version of Bloom's Taxomy

Our extensive teaching experience led us, gradually changing activities outside and inside the classroom. We initially led to record short videos (Polimedia) in order to facilitate the study and clarify questions that arise most commonly our students [10, 11, 12]. This set of videos was later given an OCW format [13]. In it, texts and applets are included in order to improve their skills in scientific language, as concepts presented in the video are also written. Later, in class, we combine the active lecture (where the student is involved with questions and / or answers), with activities that required the use of OCW. Shortly after, our university (UPV) embarked on the project to generate MOOCs where we participate actively [14]. Our published MOOC [15] was the final push to implement the flip teaching methodology, relying on the subject OCW and MOOC made.

D. What is flip teaching?

The methodology known as flipped classroom, where students have access before class to videos and interactive lessons created by teachers can turn the classroom into a place to solve problems, advance concepts, and participate in collaborative learning [16, 17, 18].

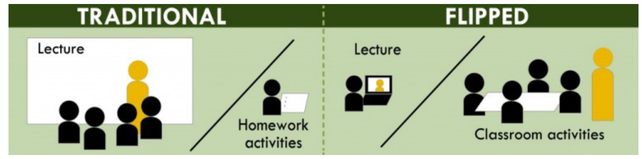

Fig. 3: Flipped Classroom

The difference with a more traditional approach lies mainly in that the acquisition of information is done by the student independently at their own pace, from the teaching materials given by the teacher. Consequently, part of the teachers work is to design activities for both a previous study and to be done in class, which also plays an important role as facilitator and driver of the teaching-learning process. In this way, teacher-students interaction increases in the classroom, to facilitate the joint development of generic and specific skills (see Fig. 3).

The methodology that we have conducted to reinforce learning is based in two key issues: student's daily work at home and a continuous feedback at the classroom with the professor and the classmate.

For all this matter, it has been essential for the students to have a detailed schedule with all the proposed tasks and activities, to dispose of good tools of communication with us, and to access to teaching materials and information quickly and orderly.

In our case, the online training materials that we use in the course is the OCW course "Estructuras Matemáticas para la Informática II", accessible from the website of the UPV [3] and MOOC "Aplicaciones de la Teoría de Grafos a la vida real" (Applications of Graph Theory to real life), active in the UPVX and EDX platform [15] (see Fig. 4). The tutorials that plan the work of students in each classroom session are available on the learning platform of the UPV, PoliformaT, member of Sakai project, where teachers and students share information.

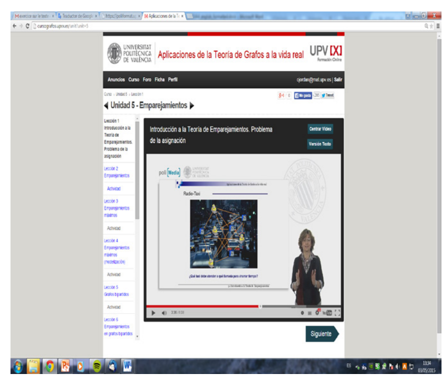

Fig.4. Screenshot of MOOC [18]

IV. Flip Teaching with MOOCs and OCWs

After application of the flip methodology over several academic years, we have come to the conclusion that despite it promotes a more meaningful learning, it has drawbacks that must be taken into account, in order to get a successful implementation. On the one hand, if the group is large it is more difficult to implement this methodology and to achieve active and ongoing collaboration of each student. The most common factors that hinder this collaboration, in our experience, are:

- Our students are not accustomed to the continuous assessment
- The workload of all subjects
- Extracurricular activities that hinder the first contact with the concepts that will be discussed in the classroom

Moreover, any innovation in the teaching-learning process will have, at the beginning, a rejection by students, perhaps motivated by a different learning style to the prevailing since childhood.

Regarding the teacher, we emphasize the effort of developing appropriate educational material, either text or video. In addition, planning is a cornerstone of this methodology so it is essential to have tutorials for monitoring. In its writing we must pay special attention to both the suitable timing of the contents as to the workload at home: reviewing knowledge, view videos, solving exercises, etc.

On the other hand, in the classroom, the teacher should be very alert to the participation of each student. A good involvement in the classroom will yield students to be more motivated to study before each session, something necessary in this methodology [19].

In the particular case of working with MOOCs, usually students find videos more attractive than text. However, we found that in some cases, depending on the level and / or learning style of the students, they prefer to consult the OCW, where the contents in text formats, applets, etc. can be found, in addition to videos.

V. Conclusions

Educational institutions have encouraged the creation of online learning materials in response to the new digital paradigm that society has experienced. Taking advantage of the accessibility features that Internet provides, several organizations have emerged as the Open Education Consortium, committed to advancing open education and its impact on global education, that envisions a world where everyone, everywhere has access to the education they need to build their futures. This is the origin of the OCW subjects, extended and complemented by MOOC courses.

In 2012 all voices pointed to the educational revolution that would assume these online courses, including voices warning about the possible disappearance of the face-to-face teaching universities as we know them today.

The statistics on enrollment, dropouts, results, show a less encouraging picture. We find some important drawbacks: many people around the world have no internet access, the problem of language, the difficulty to certificate the acquired knowledge, the lack of personal time to study (work, obligations, ...) which makes the dropout rate large, …

However, the educational structure is changing and we must take all these initiatives, adapting them to our teaching. So, flip education movement is gaining strength, the many materials now available on line, well organized and structured (OCWs and MOOCs) to facilitate the pre-study class, allowing the classroom to become a forum for debate, discussion and presentation of ideas.

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UX of social network Edmodo in undergraduate engineering students

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Abstract — **The main objective of this research is to describe the use that students make of an academic SNS (social network service) and detail the relationship between socio-demographic and academic factors associated with the use of EDMODO and the perception of the contribution to the acquisition of skills for the future career. In the analysis of user experience, participants positively evaluated EDMODO and found that the level of satisfaction is positively associated with the academic results obtained, and negatively with perceived usefulness in terms of the impact on their grades.**

Keywords — **social networking, user experience, Technological Acceptance model (TAM), university education, EDMODO.**

I. Introduction

We face changes in the near future employment trends influenced by extreme longevity, automation, computing and globally connected world, new media ecology, and super structured organizations which makes it necessary for new and old professionals to develop specific skills for the workplace in 2020, to develop computational thinking, design-oriented mentality, cognitive load management, new knowledge about media, multidisciplinarity, critical thinking, social intelligence, innovative thinking and professionals adaptability, cross capacity and virtual collaboration (Davies et al., 2011).

The concern about the development of these skills is perhaps one of the reasons that has led some educators to reap the benefits that Web 2.0 offers to transform the teaching and learning as part of the natural development of today's world, in which divisions and barriers are becoming more diffuse and which tends more to a sense of collaboration than to competition. A suitable environment for Web 2.0 is what brings and transcends of constructivism and social learning that favors the use of Learning 2.0, cheaper costs, expanding the possibility of interaction, contribution to the development of skills required to globally, among others. In this context, the most important tools in the educational space are Wikipedia, Blogs, MOOCs, e-portfolio and SNSs.

Given the potential offered by SNSs in the educational context, it is useful to analyze how they are perceived by students, which factors affect this perception, and how it relates to their academic performance. The scenario for the development of this study is the subject Business Management I, which is compulsory for all programs assigned to face undergraduate Industrial Engineering of the University of Santander students. To do this, a pedagogical practice is implemented from the concept of hybrid learning environment, keeping the requirements of a classroom course, but including educational EDMODO social network as a mechanism to facilitate collaboration and interaction.

The SNSs are defined as services based on the web that allow individuals to biuld a public or semi-public profile within a bounded system, articulate a list of other users with whom they share a connection and view and traverse their list of connections and those made by others within the system. Also allow the generation of semi-persistent public comments (Boyd, Danah & Ellison, 2007).

It is undeniable the impact of SNSs in today's society, 2014, 1.59 billion people were users of these services (Statista, 2015). In Colombia 77% of the population has access to internet (Ipsos, 2012), 99% of Colombian youth between 15 and 17% use them and the 84% for between 18 and 24 years. The most popular SNS is Facebook 100% and 92% followed by Twitter 66% and 69%, YouTube 55% and 67%, Linkedin 0% and 30%, respectively (E-marketing, nd).

So we can infer that almost all Colombian university students are SNSs users, and like colleges all over the world, belong to the "Gen Y" (born after 1982) are considered adept at "multitasking", in the teamwork, in the scanning and easily navigation in technology; they are multimedia and interactive. Although these students have a more natural relationship with technology than adults, it is clear that young disadvantaged sectors have very low technological skills despite computer technology courses received in school (Cabra Torres & Martial Vivas , 2009).

II. Theoretical review

A. The ratio of Colombian students with ICT

Cortés & Carbonell (2014) point out that Colombian University is generally characterized by a controlled use of internet connection with an average of 3.63 hours a day and social networks are the main activity carried out. The minority, corresponding to 9.7%, has a problematic use, defined as the carelessness of academic, industrial or domestic activities and their replacement by online activities, being intensive, in Colombia, the email and SNSs, founding in these students a negative correlation between kindness and responsibility. In relation to the appropriation of ICT in higher education students (Berrio & Rojas, 2014), indicate that it is the student's attitude towards ICT that determines their appropriation and application, where a strong relationship with the perception of impact is not found productive and, even when respondents had optimal conditions of access, these tools were still seen as objects of fashion and status. Finally, regarding the contribution of education to the collaborative SNSs (Bravo, Pedraza, & Herrera, 2012) it was founded that students generally perceive that Twitter encourages learning and positively contributes to the achievement of the results of the course. However, a significant percentage (30%) indicated that Twitter does not contribute to the development of collaborative work. In addition, weaknesses were observed in the dialogic communication, since 80% of the participations do not fall within the characteristics of this type of interaction.

B. The development of job skills 2020

It is considered that the educational process have to transcend the fact of training specialists to generate citizens in a society where both virtual and real network is its fundamental support then, the ability to understand how to join and build these networks, and the tools to do the purpose, the intention, protocols and standards governing the use and

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communication become increasingly important skills (Castells, 2014).

In the same line, the International Conference on Education 2012–2013, in their inter discussion, concludes that the ultimate goal of education is to form a citizen, not just a professional: a creative, entrepreneurial, critical, with high social skills, adapted to various work environments and competent in the digital world. And for this proposal, the educational process should contribute to exploiting the potential of the digital society, drawing collective intelligence from a new methodological approach linked more to interact and create content, and less just to their absorption, relying on the benefits of the turned-pedagogical use of ICT to develop skills from pedagogy with special focus on the interests of the learner, who should be formed to prevent "technological anxiety".

Taking into account comments made by the "Institute for Future" SNSs implementation of the educational process, in conjunction with appropriate teaching and learning methodologies would have a potential to develop the ability of virtual collaboration and tangentially contribution to the ability of the management of cognitive load, defined by Davies et al. (2011) as follows:

Ability to virtual communication, defined as the ability to participate, engage and motivate scattered groups that connect through technology enabling work, share ideas and be productive.

Managing the cognitive load, ability to face the avalanche of information, learning how to filter it, focusing on what is important or in the words of Castells (2014), the Internet Society, which is more complicated is not to navigate, but know where go, where to look for what you want to find and what to do with what is found, and this requires education.

C. Social networks in education

The contribution of social networks to education is still in discussion, and their advocates relate their greatest potential contribution to the generation of content (Rennie & Morrison, 2013) of which derive theoretical benefits as the provision of tools for participation, closing of gaps, development of social skills and motivation of young people to make use of shared spaces and communications, a situation that captivates them deeply. Additionally, it allows constant and economic update information if existing and known or previously explored by the student spaces are used. This position is supported by different authors.

By having tools that facilitate participation, generated by other communication schemes there are overcome barriers such as time, space and restrictive elements as cultural norms, behavioral, gender, among others (Bulbulia, Blewett, Quilling, & Kanyiwamuyu, 2010). It also facilitates the creation of community, which in the current context is given more based on affinities and less on geographical proximity as traditionally occurred. These tools also extend the traditional classroom, allowing users to join with others with similar educational interests (Pollara & Zhu, 2011) and nurturing interactions and varied typology documents and communicate more interactively combining both the sound, video and documents (Hernandez, 2008 cited in (Sarsa, 2014). Also tutoring is expanded, ensuring flexibility and ease of use by mentors and mentees promoting their success, given that the most frequent contact between mentor and mentee generates better effects (Podoll & Randall, 2005;. Dubois et al, 2002 cited in (Pollara & Zhu, 2011)).

Increasing opportunities for participation, creativity is potentially potentiated (Cabero & Marin, 2014) and a more interactive space and a dynamic learning is encouraged (Smith & Guzman, 2011 cited in Cabero & Marin, 2014), so it provides active learning and collaboration Pollara and Zhu, 2011) with benefits in increasing motivation and encouraging academic performance, from the experiential reinforcement between individual learning and collective, which contributes to the retention and skills development including multidisciplinarity and critical thinking.

Additionally, it provides computer literacy, as it is shown that not all young people are "digital natives", so efforts allowing technological appropriation and the factors that modulate this process are still required (Berrio & Rojas, 2014). It is needed to take into account the subsequent contribution to academic performance, from the spaces generated to address concerns that may facilitate monitoring by teachers, with personalized feedback encouraging the organic development of knowledge that impacts retention concepts (Rennie & Morrison , 2013).

Closing gaps contributes to the experience, that is, if teachers will use in the academic context the same instruments that students often use to communicate, the best solutions to help a better teaching molded to the characteristics of students could be identified, optimizing those variables that are the engine of learning (the emotion) and stimulating higher character skills (awareness and liberality), in addition to finding new forms of multiple literacy (Costa, Cuzzocrea, & Nuzzaci, 2014).

Regarding the development of social skills, (Valkenburg & Peter, 2009) conclude that the use of SNSs contributes positively to peer relations of adolescents and facilitates interaction with existing links (Boyd & Ellison, 2007; Haythornthwaite, 2011) where mean-face relationships and promote and collaborative work management benefits other groups as improving social relationships are inferred. Tolerance towards people of the group, extends the ability of social support, integration and cohesion in groups (Cabero & Marin, 2013), social capital formation and development of social trust (Ellison, Steinfeld, & Lampe, 2007; Valenzuela, Park, & Kee, 2009).

It also allows the development of communication skills to facilitate joint communication, as elements of oral communication with written ones are combined (Linne, 2014). They are scenes of self-expression, communication and self-advocacy, and collaborative work. Other individual benefits are derived such as the development of intrinsic motivation and self-esteem and skill acquisition of democratic leadership and participation (Cabero & Marin, 2013).

In contrast, there are positions that refute the positive impact of SNSs the learning process, and although they share the same risks contents, contact, and commercial Internet (De-Moor, Dock, Gallez, & Lenaerts, 2008), arouse an special fear in teachers and parents, as in the environment of the youth, (Douglas et al., 2008), the fact of devoting too much time to this services has negative influences on the habits and daily routines, qualifications and relationships in general. Moreover, it has come to speak of the perceived stress when it is impossible to connect the web (Labrador & Villadangos, 2010).

In relation to content, there is plenty of unfiltered and unauthenticated information and most students lack the critical skills to discern in this mass of undifferentiated material. Web 2.0 critics express concern about trust, reliability and credibility (Rennie & Morrison, 2013), as well as traceability to the power sources and fragmentation of content (Rennie & Morrison, 2013), so to know how to manage cognitive load would help positively to overcome this risk.

In contact risks and situations relating to privacy, the use of these services can have a negative impact and the destruction of the traditional roles of teacher and student (Sickler, 2007). But if using closed SNSs, this conflict will tend to zero.

Commercial risks are not observed as relevant to the object of study, but it should expand the above classification with the category of "academic risks" that may result from the continued and disruptive used of the SNSs.

This is how new technology encourages a short span of attention and takes students to demand immediate answers instead of thinking for themselves. In addition, not everyone agrees that "Millennials" are so different from their predecessors, consequently and according to this, various educational techniques should be maintained (Rennie & Morrison, 2013). In some cases it does not have a real value for learning (superficiality and informality), discouraging the development of traditional skills and competences (Pollara and Zhu, 2011).

Other authors consider that, in terms of opportunities to socialize and work within the academic processes, this need is covered by the Learning Management Systems (LMS), but studies show that participation in SNSs like Facebook is far superior to that of tools exclusively dedicated to learning processes (Pollara & Zhu, 2011).

Finally, investigations as presented by Kirschner & Karpinski (2010), Junco (2012), Karpinski, Kirschner, Ozer, Mellott, & Ochwo (2013) found a negative relationship between academic achievement (GPA) and the use of Facebook, a relationship between the promotion of the use of the SNSs and the "Multitasking" and loss of efficiency and effectiveness in the learning process, resulting in lower average tendency to procrastinate and weaknesses in time management.

D. EDMODO

Edmodo is a social platform for education, where the interaction process works through a wall, in which resources are shared, conversations are generated, concerns are posted and attended and hierarchies are eliminated, allowing the development of communication skills and argumentative easily co-evaluation. The similarity with Facebook theoretically reduces the learning curve and to be a closed educational SNSs avoids the phenomenon of "creepy tree house."

E. MAT Model

The TAM model (Davis, 1989) is a methodology to measure the successful implementation of a tool and the factors influencing their appropriation, so it can considered as mature and highly used (Yong, 2004). It argues that the probability of acceptance of a technological tool is a function of the perceived usefulness (Perceived Usefulness-PU) and perceived ease of use (Perceived Ease of use- PEOU).

With regard to the implementation of EDMODO, investigations collect the research experiences of application of this tool (Enriquez, 2014; Holland & Muilenburg, 2011; Kongchan, 2008; Ractham & Chen, 2012; Sanders, 2012; Thien et al, 2013.)

In particular, a study becomes an important theoretical reference since it addresses the evaluation of the social network Edmodo from the Technology Acceptance Model TAM (Thongmak, 2013).

The case presented by Thongmak describes the experience of implementing EDMODO following the recommendations made by some authors address about the importance of integrating natural environments for students in the teaching-learning process, as in this case, a SNS. It analyzes factors that can limit or support the adoption of this tool to increase levels of collaboration, with the studied variables: Ease of Use, Perceived Usefulness, Characteristics of the teacher and student and intended use, concluding that the first three mentioned are the key to the appropriation of this tool factors. Additionally, in their literature review, he emphasizes on the weak ownership of SNSs in education and consequent few studies providing insight into the uses and effects of its application in academic processes.

F. Problem Statement

Throughout studies reported in the literature, a consensus on the impact of SNSs in academic settings is not reached. On the one hand, authors (Boyd & Ellison, 2007; Bulbulia et al, 2010;. Ellison et al., 2007; Haythornthwaite, 2011; Linne, 2014 Pollara & Zhu, 2011; S. Valenzuela et al, 2009;. Valkenburg & Peter, 2009) argue that the impact is positive, but literature that claims otherwise is also located. Additionally, there are few studies in the Colombian context providing insight into the uses and purposes of the application of SNSs in academic processes. For this reason, it is convenient to conduct a research in classrooms intervening and evaluating the impacts of SNSs.

The general aim of this research is to evaluate the level of satisfaction of college students by implementing EDMODO, to determine factors associated with acceptance and to identify mechanisms to effectively incorporate social tools applied to the academy. Also, to describe the use that students make on educational social network EDMODO so as to characterize users and understand how the use of SNSs can contribute to academic success and / or skills development as virtual collaboration and skills cognitive load management.

G. Methodology

Type of study	Exploratory, with quasi-experimental analysis of quantitative and qualitative information.
Techniques used	Consultation university records, observation and survey filled out with auto-tracking research.
Instruments	And standardized observation sheet with open and closed questions questionnaire.
Studied universe	Students of the subject Business Management UIS ending the semester 2013-2, 2014-1
Sample size	Control group: 47 students Intervention group: 92 students
Academic variables	Academic program, level (semester), academic Trust (cumulative average above), academic performance (final grade for the course).
Sociodemographic variables	Age, Sex, College where he studied the (public or private), Origen, and NSE (socioeconomic status in Colombia).
Variables "Gen Y" (Millennials)	Investigates the perception of the respondent against the generation gap in the teaching-learning process and contribution of ICT and SNSs in the trend and multitasking
UX variables (User Experience)	Check overall satisfaction with using Edmodo and its perceived usefulness and ease of use. Perceived Usefulness: contribution to academic results and future pro professional communication with teachers and peers and access to resources. Ease of Use: ease of management activities, solution and utility concerns.
Recollection of information	Information for the study was collected by a made up of three blocks of information questionnaire. For the first block for which was used directly to university records. For two three blocks information was collected through questionnaire addressed online.
Information analysis Methodology	Using linear regression purposeful selection of variables associated factors were evaluated user satisfaction according to the proposed model TAM.

III. Results

In technology and in different environments, it has been proved that much of the appropriation of a tool comes from the perception of satisfaction experienced by the user and that is why in the process of teaching and learning any technological intervention or change in methodology is susceptible of such reviews, to find through this level of satisfaction estimation the best practices and lessons learned.

For the purposes of assessment of the experience of users who formed part of the intervention group, a questionnaire for a total of 91 participants was developed, in terms of general and specific satisfaction (communication and management of activities), about the perception of the utility of EDMODO implemented in short term (academic results and deepening of specific knowledge of Management) and long term (contribution to vocational training and job skills 2020).

As shown in Figure 1, the experience with Edmodo can be described as positive (T2B = 82%), especially for women and for those students classified in the group of high academic achievement (average equal or superior to the group), coincident with reported by Hung & Cheng, 2013 cited in Cabero & Marin 2013).

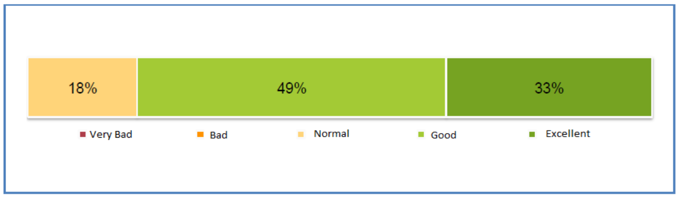

Figure 1. Marks obtained in the experience with Edmodo

On the positive side, participants felt that the tool facilitates the management of activities (T2B = 91%) and the track of their qualifications (T2B = 89%). Likewise, the ability to communicate with the teacher (T2B = 85%), ease of sharing resources in different formats (T2B = 98%) and access to interesting information on the subject (T2B = 95%) were also seen. Also with positive ratings, but not as strong, it is the overall ease of use (T2B = 74%) and the contribution to the deepening of knowledge on topics covered in the course (T2B = 76%), projecting the tool slightly positive for resolving concerns (T2B: 67%).

The positive perception towards achieving a better level of knowledge thanks to the group using EDMODO.

On the negative, the use of EDMODO is related with a low ability to communicate with peers (T2B = 55%), low contribution to the academic results (T2B = 54%) and very little contribution to vocational training (T2B = 22%).

It is perceived that EDMODO generates low ability to communicate with peers, but in the process of self-evaluation only 41% considered an active participation in the tool and in conjunction with the results in academic achievement and vocational training, it was evidenced that designated by Berrio & Rojas (2014) against the unwillingness of students to take responsibility for their own learning and performance.

The overall usability (T2B = 74%) could be affected by the work methodology. Despite this, in the case of students of Business Management I, it was found that the fact of leaving the doors open to discovery EDMODO to acquire driving skills, in some cases became a barrier to progress within the process itself.

This experience was applied to a group of students where the contribution of ICT is expected to be orientated in getting a better learning by the student and the development of some skills required for future professional work, for this reason and hand with job skills 2020 postulated by the Institute for the Future (Davies, et al., 2011), the perception of the contribution of the experience of using EDMODO was evaluated considering three skills, knowledge about new media, virtual collaboration and management of cognitive load.

The evaluation of the contribution of EDMODO to improve these skills, is very positive for rating the ability to learn different ways of working (T2B = 86%), it is also positive, but less, for Virtual Collaboration (T2B = 73%) and slightly less for the ability to know where to look for what you want to find and what to do with what is found with 63% positive mentions.

UX associations with the variables studied were reviewed for Gen Y, perceived usefulness and ease of use, finding that the love of learning from different sources is positively correlated with the grade that the user makes regarding their experience using Edmodo, a situation that may shows that curious students exploit better the diversification of opportunities they find on the road and, therefore, they appropriate them more successfully.

Related to the mechanisms and strategies of participation developed (logos) is observed a weak correlation between satisfaction and motivation that comes when getting them, a situation that may be due to occult practice by which they were delivered or not to take into account in the rating (qualification).

The TAM model maintains that the ease of use and the perceived usefulness are related to the intended use or, in this case, with the UX. When reviewing whether these associations are presented in this study, this perspective is reaffirmed as it is found that the rating of UX is positively associated with the ease of use in general, the facility to track their activities and the knowledge their progress in terms of qualifications as well as with the perceived usefulness of knowledge expressed in terms of business and the contribution to their academic performance and professional development and similarly to 2020 job skills such as Virtual Collaboration and Management of cognitive load. There is no association with the new knowledge in media and this may be related to the type of activities undertaken that generally did not required the exploration of tools but EDMODO.

As the TAM model proposes that the perception of usefulness and usability are definitive in the appropriation of a specific form of work, it is not possible to ignore that prior experience and the typical processes of resistance to change can also affect the intention of use, for this reason and using regression analysis as a technique to find out the factors associated with UX which are also related to student achievement.

The academic performance of students in the study group is related to variables such as academic confidence, age, sex, origin, Overall Rating EDMODO, NSE, Perception Rating Impact on significance levels and are indicated in Table 1.

Variable	P	sig20	sig10	sig5	sig1
Academic confidence	0,00%	*	*	*	*
Age	0,64%	*	*	*	*
Gender	1,68%	*	*	*	
Origin	3,19%	*	*	*	
UX Rate	7,98%	*	*		
Socioeconomic level	9,28%	*	*		
Perception Impact Rating	18,51%	*			

Table 1. Significant variables

Once forward the variables review process, the final model is obtained (R2 = 0.43), ie that 43% of qualifications in the grade Management I is mainly explained through the variables Academic Trust, Gender (Female) and UX (Categories 4 and 5).

In this model the variables Origin (Capital) and Perceived Impact Rating (categories 1-5) are conserved as the confounding variables of the significant variables and the overall qualification in UX, respectively.

The previous academic performance remains being the key variable in predicting the results of the students (b = 0.76), and the female continues unabated and, related to UX variables, the positive perception that the students had to EDMODO contributes about 1.5 tenths in final rating, in contrast to the consideration that the tool did not impact positively on their score. This perception subtracts more than 5/10 to academic performance (about 4/10 when the impression was positive).

Building this model (Table 2) confirms its importance in the perception of utility (impact on the rating) and usability (UX overall rating), allowing the advance assessment the success of the appropriation of the use of Web 2.0 for academic purposes to find alternatives which foster their proper implementation, in order to maximize results in the students in the short-term to adapt educational innovation projects making them more effective.

Variable	P	B	Standard error	T	Confidence interval 95%	
Academic confidence	0,0000	0,76	0,14	5,38	0,48	1,04
UX-4 score	0,1230	0,16	0,10	1,56	-0,04	0,37
UX-5 score	0,2180	0,15	0,12	1,24	-0,09	0,39
Female	0,0150	0,19	0,08	2,49	0,04	0,34
Hometown	0,0500	0,15	0,08	1,99	0,00	0,30
Impact perception Mark-1	0,1760	-0,56	0,41	-1,37	-1,38	0,26
Impact perception Mark-2	0,1650	-0,53	0,38	-1,40	-1,28	0,22
Impact perception Mark-3	0,1160	-0,53	0,33	-1,59	-1,20	0,13
Impact perception Mark-4	0,2480	-0,39	0,33	-1,16	-1,06	0,28
Impact perception Mark-5	0,1890	-0,46	0,35	-1,33	-1,16	0,23

Table 2. The model

IV. Conclussions

The use of EDMODO in the learning experience is seen as positive (T2B = 82), identifying facilitators and impeding factors in the appropriation process. The main facilitators are given in terms of communication and organization (asynchronous and easy communication with the teacher and the group in general) and in terms of organization, the tool remembers and facilitates the delivery of activities. Considering constructivist concepts free exploration was used to acquire significant learning that can be replicated to a future in adapting to other tools, additional to the management tool itself, and that this situation (knowledge in new media) became a main obstacle that prevented exploiting the potential of the tool by the typical resistance to change.

As proposed in the model TAM (Technology Acceptance Model), the variables of perceived usefulness and ease of use were found to be related to the appropriation of the tool. In relation to Edmodo users it was seen a slight but positive association (b = 0.15) between the score and results UX, where students with better academic results qualify more positively their experience in comparison with those with below average academic results. The contribution to academic results is weakly perceived (T2B = 53%) and negatively related to academic achievement (decrease of more than four tenths in qualifying the subject).

It forms part of the process of comprehensive training to prepare students for employment in the near future, in an environment characterized by change and connectivity. In this context, participants felt that the application of EDMODO helps to develop competencies such as the ability 2020 media (T2B 85%), virtual collaboration (T2B = 73%) and management of cognitive load (T2B 63 %), with positive associations with UX for the last two.

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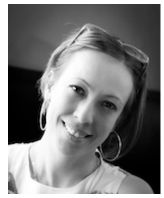

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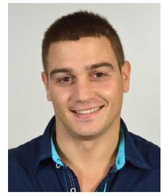

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Ball convergence for Steffensen-type fourth-order methods

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***Abstract* — We present a local convergence analysis for a family of Steffensen-type fourth-order methods in order to approximate a solution of a nonlinear equation. We use hypotheses up to the first derivative in contrast to earlier studies such as [1], [5]-[28] using hypotheses up to the fifth derivative. This way the applicability of these methods is extended under weaker hypotheses. Moreover the radius of convergence and computable error bounds on the distances involved are also given in this study. Numerical examples are also presented in this study.**

***Keywords* — Newton method, Steffensen-type methods, order of convergence, local convergence.**

I. Introduction

In this study we are concerned with the problem of approximating a locally unique solution x^* of equation

$$F(x) = 0, \tag{1}$$

where $F : D \subseteq S \to S$ is a nonlinear function, D is a convex subset of S and S is $\mathbb{R}$ or $\mathbb{C}$. Artificial intelligence and e-learning are two of the emerging needs of the information age. Authors from various other areas can follow these techniques to serve another scientific communities. Newton-like methods are famous for finding solution of (1), these methods are usually studied based on: semi-local and local convergence. The semi-local convergence matter is, based on the information around an initial point, to give conditions ensuring the convergence of the iterative procedure; while the local one is, based on the information around a solution, to find estimates of the radii of convergence balls [3, 4, 20, 21, 22, 24, 26].

Third order methods such as Euler's, Halley's, super Halley's, Chebyshev's [1]-[28] require the evaluation of the second derivative F'' at each step, which in general is very expensive. That is why many authors have used higher order multipoint methods [1]-[28]. In this paper, we study the local convergence of fourth order Steffensen-type method defined for each $n = 0,1,2,\cdots$ by

$$y_n = x_n - \frac{2F(x_n)^2}{F(x_n + F(x_n)) - F(x_n - F(x_n))}$$

$$x_{n+1} = x_n - \frac{2F(x_n)^2}{F(x_n + F(x_n)) - F(x_n - F(x_n))}\,\frac{F(y_n) - F(x_n)}{2F(y_n) - F(x_n)}, \tag{2}$$

where x_0 is an initial point. Method (2) was studied in [11] under hypotheses reaching upto the fifth derivative of function F.

Other single and multi-point methods can be found in [2, 3, 20, 25] and the references there in. The local convergence of the preceding methods has been shown under hypotheses up to the fifth derivative (or even higher). These hypotheses restrict the applicability of these methods. As a motivational example, let us define function f on $D = [-\frac{1}{2}, \frac{5}{2}]$ by

$$f(x) = \begin{cases} x^3 \ln x^2 + x^5 - x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Choose $x^* = 1$. We have that

$$f'(x) = 3x^2 \ln x^2 + 5x^4 - 4x^3 + 2x^2,\ f'(1) = 3,$$

$$f''(x) = 6x \ln x^2 + 20x^3 - 12x^2 + 10x$$

$$f'''(x) = 6 \ln x^2 + 60x^2 - 24x + 22.$$

Then, obviously, function f''' is unbounded on D. In the present paper we only use hypotheses on the first Fréchet derivative. This way we expand the applicability of method (2).

The rest of the paper is organized as follows: Section 2 contains the local convergence analysis of methods (2). The numerical examples are presented in the concluding Section 3.

II. Local convergence for method (2)

We present the local convergence analysis of method (2) in this section. Let $U(v,\rho), \overline{U}(v,\rho)$ stand for the open and closed balls in S, respectively, with center $v \in S$ and of radius $\rho > 0$.

Let $L_0 > 0, L > 0, M_0 > 0, M > 0$ and $\alpha > 0$ be given parameters. It is convenient for the local convergence analysis of method(2) that follows to define some function on the interval $[0, \frac{1}{L_0})$ by

$$g(t) = \frac{Lt}{2(1 - L_0 t)},$$

and parameters

$$r_A = \frac{2}{2L_0 + L} < \frac{1}{L_0},$$

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$$r_0 = \frac{1}{(1+\frac{M}{2})L_0} < \frac{1}{L_0}.$$

Notice that if:

$$M_0L_0 < L \Rightarrow r_A < r_0$$

$$M_0L_0 = L \Rightarrow r_A = r_0$$

$$M_0L_0 > L \Rightarrow r_0 < r_A.$$

We have that $g(r_A) = 0,$ and

$$0 \le g(t) < 1 \; foreach\, t \in [0, r_A).$$

Define function g_1 on the interval $[0, r_0)$ by

$$g_1(t) = \frac{L}{2(1-L_0t)}[1+\frac{2\alpha M_0 M^2 t}{1-(1+\frac{M_0}{2})L_0t}]t$$

and set

$$h_1(t) = g_1(t) - 1.$$

We get that $h_1(0) = -1 < 0$ and $h_1(t) \to +\infty$ as $t \to r_0^-$. It follows from the Intermediate Value theorem that function h_1 has zeros in the interval $(0, r_0)$. Denote by r_1 the smallest such zero. Moreover, define function on the interval $[0, r_0)$ by

$$p(t) = [L_0 g_1^2(t) + \frac{4M_0M}{1-(1+\frac{M_0}{2})L_0t} + \frac{L_0}{2}]t$$

and set

$$h(t) = p(t) - 1.$$

Then, we have that $h(0) = -1 < 0$ and $h(t) \to +\infty$ as $t \to r_0^-$. Hence, function h has a smallest zero $r_p \in (0, r_0)$. Furthermore, define functions on the interval $[0, r_0)$ by

$$p_1(t) = \frac{2M_0Mt^2}{1-(1+\frac{M_0}{2})L_0t},$$

$$g_2(t) = \frac{1}{2(1-L_0t)}[L + \frac{2M^2\alpha(LM_0p_1(t) + 2M^2g_1(t))t}{(1-(1+\frac{M_0}{2})L_0t}) (1-p(t))]t$$

and set

$$h_2(t) = g_2(t) - 1.$$

Then, we have $h_2(0) = -1 < 0$ and $h_2(t) \to +\infty$ as $t \to r_0^-$. Hence, function h_2 has a smallest zero $r_2 \in (0, r_0)$. Set

$$r = \min\{r_1, r_2, r_p\}. \tag{1}$$

Then, we get that for each $t \in [0, r)$

$$0 \le g_1(t) < 1, \tag{2}$$

$$0 \le p(t) < 1, \tag{3}$$

$$0 \le p_1(t) \tag{4}$$

and

$$0 \le g_2(t) < 1. \tag{5}$$

Next, using the above notation we present the local convergence analysis of method (2).

THEOREM 2.1 *Let $F: D \subseteq S \to S$ be a differentiable function. Suppose that there exist $x^* \in D$, $\alpha > 0, L_0 > 0, L > 0, M_0 > 0$ and $M > 0$ such that for each $x, y \in D$ the following hold*

$$F(x^*) = 0, F'(x^*) \ne 0, \; with \; \|F'(x^*)\| \le \alpha, \tag{6}$$

$$| F'(x^*)^{-1}(F'(x) - F'(x^*)) | \le L_0 | x - x^* |, \tag{7}$$

$$| F'(x^*)^{-1}(F'(x) - F'(y)) | \le L | x - y |, \tag{8}$$

$$| F'(x) | \le M_0, \tag{9}$$

$$| F'(x^*)^{-1} F'(x) | \le M \tag{10}$$

and

$$\overline{U}(x^*, (1+M_0)r) \subseteq D, \tag{11}$$

where r is defined by (1). Then, the sequence $\{x_n\}$ generated by method (2) for $x_0 \in U(x^*, r) - \{x^*\}$ is well defined, remains in $U(x^*, r)$ for each $n = 0,1,2,\cdots$ and converges to x^*. Moreover, the following estimates hold for each $n = 0,1,2,\cdots,$

$$| y_n - x^* | \le g_1(| x_n - x^* |) | x_n - x^* | < | x_n - x^* | < r, \tag{12}$$

and

$$| x_{n+1} - x^* | \le g_2(| x_n - x^* |) | x_n - x^* | < | x_n - x^* |, \tag{13}$$

where the "g" functions are defined above Theorem 2.1. Furthermore, if that there exists $T \in [r, \frac{2}{L_0})$ such that $\overline{U}(x^*, T) \subset D$, then the limit point x^* is the only solution of equation $F(x) = 0$ in $\overline{U}(x^*, T)$.

Proof. We shall use induction to show estimates (12) and (13). Using the hypothesis $x_0 \in U(x^*, r) - \{x^*\}$, the definition of r and (7) we get that

$$| F'(x^*)^{-1}(F'(x_0) - F'(x^*)) | \leq L_0 | x_0 - x^* | < L_0 r < 1. \quad (14)$$

It follows from (14) and the Banach Lemma on invertible functions [3, 4, 19, 20, 22, 23] that $F'(x_0)$ is invertible and

$$| F'(x_0)^{-1} F'(x^*) | \leq \frac{1}{1 - L_0 | x_0 - x^* |} < \frac{1}{1 - L_0 r}. \quad (15)$$

We can write by (6) that

$$F(x_0) = F(x_0) - F(x^*) = \int_0^1 F'(x^* + \theta(x_0 - x^*)(x_0 - x^*) d\theta. \quad (16)$$

Then, we have by (9), (10) and (16) that

$$| F(x_0) | \leq | \int_0^1 F'(x^* + \theta(x_0 - x^*)(x_0 - x^*) d\theta |$$

$$\leq M_0 | x_0 - x^* | \quad (17)$$

and

$$| F'(x^*)^{-1} F(x_0) | \leq | \int_0^1 F'(x^*)^{-1} F'(x^* + \theta(x_0 - x^*)(x_0 - x^*) d\theta |$$

$$\leq M | x_0 - x^* | \quad (18)$$

where we used $| x^* + \theta(x_0 - x^*) - x^* | = \theta | x_0 - x^* | < r$ for each $\theta \in [0,1]$. We also have by (17) and (11) that

$$| x_0 \pm F(x_0) - x^* | \leq | x_0 - x^* | + | F(x_0) |$$

$$\leq | x_0 - x^* | + M_0 | x_0 - x^* | < (1 + M_0) r,$$

so $x_0 \pm F(x_0) \in D$. Next we shall show that $F(x_0 + F(x_0)) - F(x_0 - F(x_0))$ is invertible. Using the definition of r_0, (7) and (17), we get in turn that

$$| F'(x^*)^{-1}[F(x_0 + F(x_0)) - F(x_0 - F(x_0)) - F'(x^*)] |$$

$$= | \int_0^1 [F'(x^*)^{-1}[F'(x_0 - F(x_0) + 2\theta F(x_0)) - F'(x^*)] d\theta |$$

$$\leq L_0 [| x_0 - x^* | + \int_0^1 | 1 - 2\theta || F(x_0) | d\theta]$$

$$\leq L_0 [| x_0 - x^* | + \frac{M_0}{2} | x_0 - x^* |]$$

$$= L_0 (1 + \frac{M_0}{2}) | x_0 - x^* | < L_0 (1 + \frac{M_0}{2}) r_0 < 1. \quad (19)$$

It follows from (19) that $F(x_0 + F(x_0)) - F(x_0 - F(x_0)$ is invertible and

$$| (F(x_0 + F(x_0)) - F(x_0 - F(x_0))^{-1} F'(x^*) | \leq \frac{1}{1 - L_0 (1 + \frac{M_0}{2}) | x_0 - x^* |}$$

$$< \frac{1}{L_0 (1 + \frac{M_0}{2}) r}. \quad (20)$$

Hence, y_0 is well defined by the first substep of method (2) for $n = 0$. Then, we can write

$$y_0 - x^* = x_0 - x^* - \frac{F(x_0)}{F'(x_0)} + \frac{F(x_0)}{F'(x_0)} - \frac{2F(x_0)}{F(x_0 + F(x_0)) - F(x_0 - F(x_0))}$$

$$= -[F'(x_0)^{-1} F'(x^*)][\int_0^1 F'(x^*)^{-1}[F(x^* + \theta(x_0 - x^*)) - F'(x_0)]$$

$$\times (x_0 - x^*) d\theta] + \frac{\Gamma}{\Gamma_1} \quad (21)$$

where

$$\Gamma := 2(F'(x^*)^{-1} F'(x_0))^2 [\int_0^1 F'(x^*)^{-1}(F'(x_0 - F(x_0) + 2\theta F(x_0)) - F'(x_0)] F'(x^*) d\theta$$

and

$$\Gamma_1 := [F'(x^*)^{-1} F'(x_0)][F'(x^*)^{-1}(F(x_0 + F(x_0)) - F(x_0 - F(x_0))].$$

The first expression at the right hand side of (21), using (8) and (15) gives

$$| F'(x_0)^{-1} F'(x^*)][\int_0^1 F'(x^*)^{-1}[F(x^* + \theta(x_0 - x^*)) - F'(x_0)](x_0 - x^*) d\theta |$$

$$\leq \frac{L | x_0 - x^* |}{2(1 - L_0 | x_0 - x^* |)}. \quad (22)$$

Using (6), (8), (17) and (18) the numerator of the second expression in (21) gives

$$| 2(F'(x^*)^{-1} F'(x_0))^2 [\int_0^1 F'(x^*)^{-1}(F'(x_0 - F(x_0) + 2\theta F(x_0)) - F'(x_0)] F'(x^*) d\theta |$$

$$\leq 2\alpha M^2 | x_0 - x^* |^2 L \int_0^1 | 1 - 2\theta | d\theta | F(x_0) |$$

$$\leq M^2 M_0 \alpha L | x_0 - x^* |^3 . \quad (23)$$

Then, it follows from (2), (15), (20), (21)-(23) that

$$|y_0 - x^*| \le \frac{L|x_0 - x^*|^2}{2(1 - L_0|x_0 - x^*|)}$$

$$+ \frac{2\alpha L M_0 M^2 |x_0 - x^*|^3}{2(1 - L_0|x_0 - x^*|)(1 - (1 + \frac{M_0}{2})L_0|x_0 - x^*|)}$$

$$= g_1(|x_0 - x^*|)|x_0 - x^*| < |x_0 - x^*| < r,$$

which shows (12) for $n = 0$ and $y_0 \in U(x^*, r)$. Next, we shall show that $2F(y_0) - F(x_0)$ is invertible. First notice that by the first substep of method (2) for $n = 0$, (9), (10), (20) and the definition of function p_1 we have that

$$|y_0 - x_0| = 2\left|\frac{F'(x^*)^{-1}F(x_0)F(x_0)}{F'(x^*)^{-1}(F(x_0 + F(x_0)) - F(x_0 - F(x_0))}\right|$$

$$\le \frac{2M_0 M|x_0 - x^*|^2}{1 - (1 - (1 + \frac{M_0}{2})L_0|x_0 - x^*|)} = p_1(|x_0 - x^*|). \tag{24}$$

Then, using the definition of function $p, x_0 \neq x^*$, (3), (4), (7), (12) (for $n = 0$) and (24), we get in turn that

$$|(F'(x^*)(x_0 - x^*))^{-1}[2F(y_0) - F(x_0) - F'(x^*)(x_0 - x^*)]|$$

$$\le |x_0 - x^*|^{-1}|[2|F'(x^*)^{-1}[F(y_0) - F(x^*) - F'(x^*)(y_0 - x^*)]|$$

$$+ 2|y_0 - x_0| + |F'(x^*)^{-1}[F(x_0) - F(x^*) - F'(x^*)(x_0 - x^*)]|]$$

$$\le |x_0 - x^*|^{-1}\,[L_0|y_0 - x^*|^2 + 2p_1(|x_0 - x^*|)$$

$$+ \frac{L_0}{2}|x_0 - x^*|^2]$$

$$\le [L_0 g_1^2(|x_0 - x^*|) + \frac{4M_0 M}{1 - (1 - (1 + \frac{M_0}{2})L_0|x_0 - x^*|)} + \frac{L_0}{2}]|x_0 - x^*|$$

$$= p(|x_0 - x^*|) < 1. \tag{25}$$

It follows from (25) that $2F(y_0) - F(x_0)$ is invertible and

$$|(2F(y_0) - F(x_0))^{-1}F'(x^*)| \le \frac{1}{1 - p(|x_0 - x^*|)}. \tag{26}$$

Hence, x_1 is well defined by the second step of method (2) for $n = 0$. We can also write that

$$x_1 - x^* = x_0 - x^* - \frac{F(x_0)}{F'(x_0)} + \frac{F(x_0)}{F'(x_0)} - \frac{2F(x_0)^2}{F(x_0 + F(x_0)) - F(x_0 - F(x_0))}$$

$$\times \frac{F(y_0) - F(x_0)}{2F(y_0) - F(x_0)}$$

$$= x_0 - x^* - \frac{F(x_0)}{F'(x_0)}$$

$$+ \frac{N}{\Gamma_2} \tag{27}$$

where

$$F'(x^*)^4 N = F(x_0)(2F(y_0) - F(x_0))(F(x_0 + F(x_0)) - F(x_0 - F(x_0))$$

$$-2F(x_0)F'(x_0)(F(y_0) - F(x_0)))$$

$$= 2F(x_0)^2\{\int_0^1 [F'(x_0 - F(x_0) + 2\theta F(x_0)) - F'(x_0)](F(y_0) - F(x_0))d\theta$$

$$+ \int_0^1 F'(x_0 - F(x_0) + 2\theta F(x_0))F(y_0)d\theta\} \tag{28}$$

and

$$\Gamma_2 := (F'(x^*)^{-1}F'(x_0))F'(x^*)^{-1}(F(x_0 + F(x_0)) - F(x_0 - F(x_0))F'(x^*)^{-1}(2F(y_0) - F(x_0)).$$

Using (9), (17), (18), (24) and (28), we get that

$$|N| \le 2|F'(x^*)||F'(x^*)^{-1}F(x_0)|^2\,\{|\int_0^1 F'(x^*)^{-1}[F'(x_0 - F(x_0)$$

$$+ 2\theta F(x_0)) - F'(x_0)]d\theta|$$

$$\times|\int_0^1 F'(x^*)^{-1}F'(x_0 - F(x_0) + 2\theta F(x_0))(y_0 - x_0)d\theta|$$

$$+ \int_0^1 F'(x^*)^{-1}F'(x_0 - F(x_0) + 2\theta F(x_0))d\theta\,||F'(x^*)^{-1}F(y_0)\}$$

$$\le 2\alpha M^2|x_0 - x^*|^2\,[\frac{LM_0}{2}|x_0 - x^*||y_0 - x^*| + M^2|y_0 - x^*|]$$

$$\le \alpha M^2|x_0 - x^*|^2\,[LM_0|x_0 - x^*|\,p_1(|x_0 - x^*|) + 2M^2 g_1(|x_0 - x^*|)|x_0 - x^*|]$$

$$\le \alpha M^2(LM_0 p_1(|x_0 - x^*|) + 2M^2 g_1(|x_0 - x^*|))|x_0 - x^*|^3. \tag{29}$$

Then, using (5), (15), (20), (22) and (26)-(30), we get that

$$|x_1 - x^*| \le \frac{L|x_0 - x^*|^2}{2(1 - L_0|x_0 - x^*|)}$$

$$+ \frac{2\alpha M^2[LM_0 p_1(|x_0 - x^*|) + 2M^2 g_1(|x_0 - x^*|)]|x_0 - x^*|^3}{2(1 - L_0|x_0 - x^*|)(1 - (1 + \frac{M_0}{2})L_0|x_0 - x^*|)(1 - p(|x_0 - x^*|))}$$

$$= g_2(|x_0 - x^*|)|x_0 - x^*| < |x_0 - x^*| < r,$$

which shows (13) for $n = 0$ and $x_1 \in U(x^*, r)$. By simply replacing

x_0, y_0, x_1 by x_k, y_k, x_{k+1} in the preceding estimates we arrive at estimates (12) and (13). Using the estimate $|x_{k+1} - x^*| < |x_k - x^*| < r$, we deduce that $x_{k+1} \in U(x^*, r)$ and $\lim_{k\to\infty} x_k = x^*$. To show the uniqueness part, let $Q = \int_0^1 F'(y^* + \theta(x^* - y^*))d\theta$ for some $y^* \in \overline{U}(x^*, T)$ with $F(y^*) = 0$. Using (6) we get that

$$|F'(x^*)^{-1}(Q - F'(x^*))| \le \int_0^1 L_0 |y^* + \theta(x^* - y^*) - x^*| d\theta$$

$$\le \int_0^1 (1-\theta)|x^* - y^*| d\theta \le \frac{L_0}{2} R < 1. \qquad (30)$$

It follows from (30) and the Banach Lemma on invertible functions that Q is invertible. Finally, from the identity $0 = F(x^*) - F(y^*) = Q(x^* - y^*)$, we deduce that $x^* = y^*$. ▪

REMARK 2.2

1. In view of (8) and the estimate

$$\| F'(x^*)^{-1} F'(x) \| = \| F'(x^*)^{-1}(F'(x) - F'(x^*)) + I \|$$

$$\le 1 + \| F'(x^*)^{-1}(F'(x) - F'(x^*)) \| \le 1 + L_0 \| x - x^* \|$$

condition (10) can be dropped and M can be replaced by $M(t) = 1 + L_0 t$.

2. The results obtained here can be used for operators F satisfying autonomous differential equations [3] of the form

$$F'(x) = P(F(x))$$

where P is a continuous operator. Then, since $F'(x^*) = P(F(x^*)) = P(0)$, we can apply the results without actually knowing x^*. For example, let $F(x) = e^x - 1$. Then, we can choose: $P(x) = x + 1$.

3. The radius r_A was shown by us to be the convergence radius of Newton's method [2]-[4]

$$x_{n+1} = x_n - F'(x_n)^{-1} F(x_n) \text{ foreach } n = 0, 1, 2, \cdots \qquad (31)$$

under the conditions (8) and (9). It follows from the definition of r that the convergence radius r of the method (2) cannot be larger than the convergence radius r_A of the second order Newton's method (31) if $L_0 M_0 \ge L$. Even in the case $L_0 M_0 < L$, still r may be smaller than r_A. As already noted in [3, 4] r_A is at least as large as the convergence ball given by Rheinboldt [25]

$$r_R = \frac{2}{3L}. \qquad (32)$$

In particular, for $L_0 < L$ we have that

$$r_R < r$$

and

$$\frac{r_R}{r_A} \to \frac{1}{3} \text{ as } \frac{L_0}{L} \to 0.$$

That is our convergence ball r_A is at most three times larger than Rheinboldt's. The same value for r_R was given by Traub [26].

4. It is worth noticing that method (2) is not changing when we use the conditions of Theorem 2.1 instead of the stronger conditions used in [1, 5, 11-28]. Moreover, we can compute the computational order of convergence (COC) defined by

$$\xi = \ln\left(\frac{\| x_{n+1} - x^* \|}{\| x_n - x^* \|}\right) / \ln\left(\frac{\| x_n - x^* \|}{\| x_{n-1} - x^* \|}\right)$$

or the approximate computational order of convergence

$$\xi_1 = \ln\left(\frac{\| x_{n+1} - x_n \|}{\| x_n - x_{n-1} \|}\right) / \ln\left(\frac{\| x_n - x_{n-1} \|}{\| x_{n-1} - x_{n-2} \|}\right).$$

This way we obtain in practice the order of convergence in a way that avoids the bounds involving estimates using estimates higher than the first Fréchet derivative of operator F.

III. NUMERICAL EXAMPLES

We present numerical examples in this section.

EXAMPLE 3.1 *Let* $D = [-\infty, +\infty]$. *Define function* f *of* D *by*

$$f(x) = \sin(x). \qquad (1)$$

Then we have for $x^* = 0$ that $L_0 = L = M = M_0 = 1, \alpha = 1$. The parameters are given in Table 1.

$r_A = 0.6667$
$r_0 = 0.6667$
$r_1 = 0.4000$
$r_p = 0.1138$
$r_2 = 0.2240$
$\xi_1 = 4.9901$

Table 1

EXAMPLE 3.2 *Let* $D = [-1, 1]$. *Define function* f *of* D *by*

$$f(x) = e^x - 1. \qquad (2)$$

Using (2) and $x^* = 0$, we get that $L_0 = e - 1 < L = M = M_0 = e, \alpha = 1$. The parameters are given in Table 2.

$r_A = 0.3249$
$r_0 = 0.2467$
$r_1 = 0.0967$
$r_p = 0.0262$
$r_2 = 0.0372$
$\xi_1 = 4.3370$

Table 2

EXAMPLE 3.3 *Returning back to the motivational example at the introduction of this study, we have* $L_0 = L = 146.6629073, M = 101.5578008, M_0 = 3M, \alpha = 1.$ *The parameters are given in Table 3.*

$r_A = 0.0045$
$r_0 = 4.4467e-6$
$r_1 = 0.2818$
$r_p = 0.0575$
$r_2 = 0.0001$
$\xi_1 = 3.8283$

Table 3

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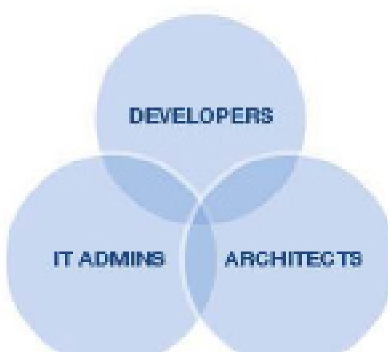

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